Analytical Model for the Characterization of the Guiding Zone Tribotest for Tube Hydroforming

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Common part failures in tube hydroforming include wrinkling, premature fracture, and unacceptable part surface quality. Some of these failures are attributed to the inability to optimize tribological conditions. There has been an increasing demand for the development of effective lubricants for tube hydroforming due to widespread application of this process. This paper presents an analytical model of the guiding zone tribotest commonly used to evaluate lubricant performance for tube hydroforming. Through a mechanistic approach, a closed-form solution for the field variables contact pressure, effective stress/strain, longitudinal stress/strain, and hoop stress can be computed. The theoretical model was validated by the finite element method. In addition to determining friction coefficient, the expression for local state of stress and strain on the tube provides an opportunity for in-depth study of the behavior of lubricant and associated lubrication mechanisms. The model can aid as a quick tool for iterating geometric variables in the design of a guiding zone, which is an integral part of tube hydroforming tooling. [DOI: 10.1115/1.3090888]

Keywords: tribotest, friction coefficient, tube hydroforming, closed-form equations

1 Introduction

Tube hydroforming (THF) is a process of manufacturing intricate shapes from tubular blanks. A hydraulic fluid is pressurized inside the tube to a yield point, hence forcing the blank to conform to the die shape. To increase shaping capability, the tubular blank ends are usually fed toward the die cavity during pressurization. Tube hydroforming has gained wide acceptance in the automotive and aerospace industries due to its advantages over stamping, such as part consolidation, weight reduction, higher part quality, fewer secondary operations, improved structural strength, and increased stiffness [1–4].

Better understanding of tribological aspects in THF is imperative for advancement of this technology. Common failures in THF include wrinkling, premature fracture, and unacceptable part surface quality [5,6]. Some of these failures are attributed to either utilizing ineffective lubrication/lubricant or failure to optimize tribological conditions in the process design. THF can be categorized into three friction zones: the guiding zone, the transition zone, and the expansion zone (Fig. 1). Research has shown that the three friction zones exhibit different states of stress [7,8]. These stress differences, which also imply different lubrication mechanisms, have led to the development of tribotests that can be used to study numerous tribological aspects in THF [8–12]. This includes (a) screening of new lubricants that can perform well in the three friction zones, (b) determination of friction coefficient via tribotest for use in numerical modeling, and (c) determination of wear characteristics of the dies used.

Three variants of tribotests are normally used for the guiding zone, as shown in Fig. 2. All of these tests involve pressurizing the tubular specimen to the required pressure level and pushing the tube through a cylindrical die.

Variant I was originally developed at the University of Darmstadt in collaboration with Schuler [8]. In this test, the normal load is measured via a load cell connected to the upper half of the die. Applying Coulomb’s law, the interface friction coefficient can be determined by Eq. (1). Friction force, \( F_f \), and normal load, \( F_C \), are both measured by load cells connected to the system. To facilitate measurement of \( F_C \), split die system is used.

\[
\mu = \frac{F_f}{F_C} \tag{1}
\]

This test requires careful design of the tooling to ensure that energy flow in the system components does not result in significant errors in the measurement of the normal load.

Variant II was originally developed at the Engineering Research Center for Net Shape Manufacturing at the Ohio State University [13,14]. In this test, the vertical shaft that holds the tube is connected to a load cell that measures the frictional force, \( F_f \). Unlike variant I, the normal load in this test is determined indirectly using the internal pressure of the tube and the properties of the tube material. The friction coefficient is determined by

\[
\mu = \frac{cF_f}{P_d \pi D I L} \tag{2}
\]

where \( F_f \) is the friction force, \( P_d \) is the internal pressure in the tube, \( D_i \) is the internal diameter of the tube, \( L \) is the effective length of the tube, and \( c \) is a constant. Accurate determination of \( c \) is critical. For higher pressure and tubular specimens with higher diameter-to-wall thickness ratios, the value of \( c \) will approach 1.

Variant III, developed at the University of Paderborn [15,16], differs from variant II in how the friction force is determined. In this test, two punches are connected to the tube ends. These punches are connected to separate load cells to measure the friction force. This test allows emulation of the compression of a tube being pushed through a die. The friction force is determined by taking the difference in the friction loads measured by the two load cells. The friction coefficient can be determined by Eq. (3). The test assumes that the internal pressure is equal to the pressure...
at the tool-workpiece interface. In reality, the interface pressure may vary drastically depending on the friction level and geometric variables.

\[
\mu = \frac{F_1 - F_2}{P_i \pi D_i L}
\]

While all of the test variants discussed above can be used to rank lubricants effectively, they cannot accurately determine interface pressure distribution, which is a vital parameter for accurate determination of friction coefficient as well as for studying other tribological aspects at the interface, such as the wear rate of the tools.

2 Objectives and Approach

The objectives of this paper are to (a) establish a closed-form solution that describes the principal stress state and strain state of a tube that is pushed through a die under internal pressure loading, and (b) formulate a closed-form solution that will facilitate determination of interface friction assuming Coulomb’s law. The derived equations should also be valuable in designing guiding zones where variations in the interface pressure loading are a function of interface friction and the ratio between tube length and tube diameter. The analytical model will also bring about better understanding of the severity of deformation at the tool-workpiece interface as the state of stress/strain varies along the tube length.

The derivation commences by establishing expressions for the principal stresses and strains acting on a plastically deformed tubular material that is being pushed along a die under constant internal pressure \( P_i \). These are radial stress \( \sigma_r \), longitudinal stress \( \sigma_z \), hoop stress \( \sigma_h \), longitudinal strain \( \varepsilon_z \), and radial strain \( \varepsilon_r \). Hoop strain, \( \varepsilon_h \), is assumed to be zero because the tube is constrained by the die. Effective stress and effective strain are established based on von Mises yield criteria, and the deforming tubular material is assumed to follow the power law, \( \bar{\sigma} = K \bar{\varepsilon}^n \), where \( \bar{\sigma} \) is the flow stress, \( K \) is the strength coefficient, \( \bar{\varepsilon} \) is the effective strain, and \( n \) is the strain hardening exponent. After the state of stress is established, the expression for interface friction assuming Coulomb’s law is derived. The analytical model is verified by a comparison with finite element results. Finally, the potential applications of the developed analytical model are discussed.

3 Stress and Strain Analysis of the Guiding Zone Tribotest

3.1 Assumptions. The loading conditions in the guiding zone where the material is fed toward the die cavity are dictated by the internal pressure and axial feeding. The axial feeding is generally done at a relatively lower fluid pressure as compared with the calibration pressure, which is activated at the end of the THF process. The loading process in the guiding zone can be considered to be approximately proportional along the tube length. Thus, Hencky’s deformation theory can be applied in the analysis. It has been shown that under proportional loading, the magnitudes of resulting strains are path independent and reduce the governing equations to Hencky’s total deformation theory. Researchers have also shown that the deformation theory of plasticity may be used for a range of loading paths other than the proportion loading without violating the general requirements for physical soundness of plasticity theory.

Varma et al. found that by changing the loading conditions during hydroforming, one can subject the tube to nonproportional or proportional strain paths. In their studies, they found that non-

![Fig. 1](image1.png)

*(a) Friction zone in THF; (b) typical THF tooling*

![Fig. 2](image2.png)

*Variants of guiding zone tribotests*
theory is adopted. Various simplifications and assumptions used throughout the derivations are as follows.

- The tube is considered as a thin wall structure.
- Longitudinal stress through the thickness direction is assumed to be uniform.
- Shear stress through the thickness direction induced by friction force is neglected.
- The tube is considered as a thin wall structure.
- Throughout the process, the material is considered to be in plastic state of deformation.

### 3.2 Stress Analysis

The scheme of the test is shown in Fig. 3. Figure 4 shows the stress state on the element cut from the tube wall. In the guiding zone tribotest, the tube is first pressurized. When the yielding pressure is attained, the tube expands until it establishes contact with the die. The gap between the die and the tube before pressurization starts is of the order of 1% of the tube diameter.

Since the tube is pushed against a frictional surface, the internal pressure $P_i$ has forced the tube material to conform to the die surface such that strain in the hoop direction $e_r=0$, achieving plane strain condition. Based on Hencky’s total deformation theory [17] and the condition $e_r=0$, Eq. (4) holds.

$$e_r = \frac{d\sigma}{d\sigma} (\sigma_r - (\sigma_z + \sigma_r)/2) = 0 \Rightarrow \sigma_r = (\sigma_z + \sigma_r)/2 \quad (4)$$

Taking the balance of forces in the $r$-$\theta$ and $r$-$z$ plane, where the element width is 1 unit, Eqs. (5) and (6) can be obtained.

$$P_r \theta + \sigma_r \rho - 2\sigma_{r\theta} \sin \frac{\theta}{2} - \theta = 0 \Rightarrow \sigma_{\theta} = (\sigma_r \rho + P_r \theta)/t \quad (5)$$

$$e_r = \mu \sigma_r, \quad \frac{d\sigma_z}{dz} = \mu \frac{\sigma_r - \sigma_z}{dz} \quad (6)$$

In the guiding zone tribotest, there are two sections along the longitudinal direction of the tube (Fig. 5).

(a) **Section I**: The section near the tube end which is pushed by a punch and characterized by the conditions that $\sigma_r < \sigma_z$ and $e_z < 0$.

(b) **Section II**: The section near the free tube end characterized by the conditions that $\sigma_r < \sigma_z$ and $e_z > 0$.

In other words, section I will exhibit compressive longitudinal stress while section II will exhibit tensile longitudinal stress. It should be noted that the lengths $l_1$ and $l_2$ will vary based on the location along the tube where $\sigma_z = \sigma_t$. Details on the determination of $l_1$ and $l_2$ are given in Sec. 3.4. Using Von Misses yield criteria, the effective stress and strain under plane strain condition, $e_{\theta}=0$, can be expressed by Eqs. (8) and (9) for sections I and II, respec-

![Fig. 3 Scheme of guiding zone tribotest](image1)

![Fig. 4 Stresses acting on the element: (a) r-\theta plane and (b) r-z plane](image2)

![Fig. 5 Stress state of tube covered by sections I and II](image3)
Equations (4), (5), (7), and (10) give the inter-relationship of stresses along the tube length. To have a complete description of the stresses and strains of section I. Substituting thickness expression into Eq. (15) and integrating it, Eq. (16) can be obtained.

\[
\frac{d\sigma_z}{dz} = \frac{d(fK\beta^{n+1}(-e_z)^n + P_i)}{dz} = \frac{r_0}{t - 1} K \beta^{n+1} r_i \left(\frac{r_0}{t} - \frac{1}{2} K \beta^{n+1} n(-e_z)^n - 1\right) dz
\]

(14)

Equation (16) can be solved numerically to obtain longitudinal strain, \( e_z \), which can be substituted into Eqs. (11) and (12) to compute longitudinal stress, \( \sigma_z \), and radial stress, \( \sigma_r \), respectively. When longitudinal strain \( e_z \) is small, it is reasonable to assume that the deformed tube thickness \( t \) approximate the original thickness \( t_0 \). Thus, Eq. (7) can be reduced to \( d\sigma_2/dz = (\mu/t)\sigma_z \). Thus the second term on the left hand side of Eq. (15) can be ignored. By substituting \( r_0/t_0 = \alpha \) and \( r_1/t_0 = \alpha - 1 \) into Eq. (15), Eq. (17) can be obtained. Integrating Eq. (17), and considering the boundary conditions that \( e_z = 0 \) at \( z = 0 \), an explicit expression for \( e_z \) can be obtained as given in Eq. (18). Details on the integration of Eq. (17) are given in Appendix C.

\[
\int_0^{e_z} \left( \frac{\alpha - 1}{2} K \beta^{n+1} \frac{1}{t_0} \right) \frac{d\sigma_z}{dz} = z
\]

(17)

By substituting Eq. (18) into Eqs. (11) and (12), the radial and longitudinal stresses can be obtained as functions of interface friction, internal pressure, wall thickness, and ratio of tube outer radius to wall thickness. Equation (19) gives a complete description of the stresses and strains of section I.

\[
\sigma_r = -\left( f K \beta^{n+1} (-e_z)^n + P_i \right)
\]

(13)

where \( r_i/t = (r_0 - t)/t \), \( \lambda = r_i/t \), and \( f = (\lambda - 1)/(\lambda - 1) \).

As the tube is pushed through the die, the variable \( t \) (thickness of tube) varies along the \( z \) axis during deformation. Figure 6 shows the change in parameter \( f \) with respect to \( \lambda \). It can be observed that when \( \lambda = 5 \), and \( f = 1.125 \). As \( \lambda \) increases beyond 5, \( f \) decreases asymptotically to 1. For values of \( \lambda \) greater than 5, the variation in parameter \( f \) can be ignored. Thus, \( f \) can be taken as a constant in the longitudinal direction.

(14) is obtained by taking a derivative of \( \sigma_r \) pertaining to Eq. (13) with respect to \( dz \). Substituting Eqs. (7), (11), and (12) into Eq. (14), Eq. (15) can be obtained. With axial load, the tube thickness \( t \) will increase and can be computed as \( t = t_o e^{-ez} \) from volume constant condition.
theoretically be greater than 

By the same derivation process, Eq. (20) can be obtained to describe the stresses and strains of section II. The detailed derivation of Eq. (20) is given in Appendix A.

\[ \sigma_s = \begin{cases} \frac{(2\alpha - 1) - \mu l}{K \beta^{\alpha+1}} p_1 \exp \left( \frac{-\mu z}{(2\alpha - 1)\ell_0} \right) & \text{for } \alpha > \sigma_r \end{cases} \]

\[ \sigma_z = \begin{cases} 1 - \mu \ell_0 & \text{for } \alpha = 0 \end{cases} \]

By substituting Eqs. (20), \( L_2 \) can be determined.

\[ L_0 = \int_0^L e^{\frac{z}{2\alpha - 1}} dz, \quad l_2 = L, \quad l_1 = 0 \]

4 Derivation of Expression for Friction Coefficient

As discussed in Sec. 1, most of the existing tribotest variants for the guiding zone assume that the interface pressure between the tube and the die is approximately the same as the internal tube fluid pressure [16, 21]. The radial stress derived in Sec. 3 (Eqs. (19) and (20)) shows clearly that the interface pressure may vary drastically depending on the coefficient of friction and other geometric variables. In this section, we will derive the expression for interface friction. This derivation will include parameters that can be obtained in the guiding zone tribotest experiments, such as friction force \( F_f \) measured by the load cell (refer to Fig. 2) and fluid pressure \( P \) supplied inside the tube, and the measured original tube length \( L_0 \) and deformed tube length \( L \).

In most of the guiding zone tribotest, the longitudinal strain is small and the radial pressures can be determined by Eqs. (19) and (20), which give the contact pressures at the tool-tube interface. The two equations represent two contact scenarios that may occur during the test. One of the contact conditions is that the tube is fully covered by section II, and the other is that the tube is covered by sections I and II. When the tube is covered by sections I and II, Eqs. (19) and (20) in both used to calculate the contact pressure. The state of stress under this condition is shown in Fig. 7. Friction force \( F_f \) is now expressed by Eq. (26). Equation (27) is obtained by substituting Eqs. (19) and (20) into Eq. (26). Substituting Eq. (25) into Eq. (28), Eq. (29) is obtained. From Eq. (29), the coefficient of friction \( \mu \) can be expressed as shown in Eq. (30).

\[ F_f = \int_0^{l_2} 2 \pi r_2 \mu l \exp \left( \frac{-\mu z}{\ell_0} \right) dz_2 \]

\[ F_f = \int_0^{l_1} 2 \pi r_2 \mu l \exp \left( \frac{-\mu z}{\ell_0} \right) dz_2 + \int_0^{l_1} 2 \pi r_2 \mu l \left( \frac{L}{L} \right) dz_1 \]

\[ V_o = V \Rightarrow 2 \pi r_2 L_0 \ell_0 = 2 \pi r_2 \int_0^L t_0 e^{\frac{z}{2\alpha - 1}} dz \]
on expressing Coulomb friction as a function of interface pressure results for both friction conditions. Figure 9 shows the comparison between the analytical model and FEA deformation. The analytical model agrees well with the FEA code, DEFORM 2D. Various case studies were simulated, two of which are presented in this paper. Figure 8 shows the FEA model used. The die and punch were treated as rigid bodies and the tube was discretized by 2000 quadrilateral elements with five elements across the wall thickness. The power law flow stress equation was used to describe the material behavior. The forming duration was set at 10 s. The die was fixed while the punch was assigned a longitudinal velocity of 15 mm/s. A pressure of 30 MPa was applied on the inner surface of the tube as internal fluid pressure. The interface friction at the die-tube interface was prescribed by assuming Coulomb friction.

Figure 9 presents state variables with two friction condition: one for friction coefficient \( \mu = 0.05 \) and another for \( \mu = 0.2 \). Figure 9 shows the comparison between the analytical model and the FEA model for contact pressure, effective stress, and effective strain distribution. The analytical model agrees well with the FEA results for both friction conditions. Figure 9(f) shows the punch load and strain. The punch load for \( \mu = 0.05 \) from analytical model is 50 kN. When friction coefficient of \( \mu = 0.2 \) was used, the load increased to 240 kN. The corresponding FEA results for the punch load are 44 kN and 210 kN for \( \mu = 0.05 \) and \( \mu = 0.2 \), respectively. These values are close to that obtained using the analytical model.

The punch load history from the finite element simulation shown in Fig. 9(f) shows two distinct stages. The punch load increases gradually with an increase in the forming time up to a maximum load; thereafter a constant load is maintained for the whole forming duration. The initial stage shows load buildup before the tube starts to slide against the die. The analytical model provides punch load at the commencement of sliding, i.e., the initial stage is not accounted in this model. It should also be noted that the load built-up duration before the tube start to slide depends on the prescribed interface friction, as can be observed in Fig. 9(f).

5.2 Potential Areas of Application for the Established Closed-Form Solutions. The established equations can provide local distribution of field variables along the tube length in terms of principal stresses and strains as a function of friction coefficient, fluid pressure, and geometric variables. These equations can facilitate better understanding of tribological aspects in the guiding zone and can be helpful in the design of tooling for THF.

Tribological aspects in THF. Most frictional data obtained from existing tribotests for the guiding zone have been determined by assuming that the internal fluid pressure is the same as the interface pressure. Figures 9(a) show that the interface pressure can increase dramatically beyond the internal fluid pressure (\( P_i \)). Figure 9(a) shows that at a friction coefficient of \( \mu = 0.2 \), the interface pressure varied from 30 MPa to 62 MPa when the internal fluid pressure was 30 MPa. While coefficient of friction may be reasonably approximated by assuming that internal fluid pressure equals interface pressure, using the internal pressure for studying other tribological aspects such as die wear may lead to significant error. Figure 9(c) shows the steep gradients for the longitudinal stress of the order of 0–500 MPa. High compressive stresses dictated by \( \sigma_c \) can provide useful information on what types of lubricants may perform, as well as giving insight on possible changes in the tube surface morphology. Furthermore, knowing the contact stress at the tool-tube interface together with longitudinal stress distributions should provide information to the tribologist/lubricant formulator on whether microplasto-hydrostatic or microplasto-hydrostatic lubrication mechanisms are likely to occur. For highly strain hardening materials, the local strain distribution may provide insight as to what types of lubricant chemistries may be applicable on surfaces that exhibit significant hardening during deformation.

As discussed in Sec. 1, most of the guiding zone tribotests developed to date assume that the interface pressure to be equal to the internal fluid pressure. From the derived equations for contact pressure, see Fig. 9(a), the approximation of internal pressure will be reasonable only when the friction level is small and when the tube sample is shorter. As seen in Fig. 9(a), up to a tube length of 75 mm, the contact pressure is equal to the internal fluid pressure when \( \mu = 0.05 \). The experimental results for the guiding zone from Hwang et al. [21] were compared with the results from the derived analytical model. Hwang et al. [21] carried out the experiments using 70 mm long tube with an outer diameter of 72 mm and a wall thickness of 3 mm. The internal pressure of 20 MPa was assumed to be equal to the contact pressure. Using lubricant R68, they obtained a friction load of 14.5 kN, which yielded a friction coefficient \( \mu = 0.046 \). From the expression given in Eq. (30), where \( \alpha = r_d/t_o = 12 \), the interface friction can be determined as follows:

\[
\mu = \frac{2\times12 - 1}{70} \times 14.500 \left( \frac{4\times36 \times 20(12 - 1)^2}{3} + 1 \right) = 0.0467
\]

The friction value obtained from the analytical expression is almost identical to what was obtained by Hwang et al. [21]. As discussed above this should be the case with low friction levels.

Tube hydroforming tooling design. The guiding zone is an integral part of THF tooling. The purpose of the guiding zone is to hold and align the tube. It is where the tube is completely sealed so that high forming pressure can be attained. The guiding zone also serves as a gateway for the material to be fed to the die cavity. Since the highest sliding velocity is encountered in this zone, the die is susceptible to failure due to wear. Figure 1(b) shows a typical THF tooling with feeding die inserts at the guiding zone. Due to high sliding velocity and friction stress, the guid-
ing zone may be subjected to severe wear. Using the established
equations, tool designers can quickly plot a “friction hill enve-
lope” to aid in determining suitable geometric parameters, load
requirements for press and axial cylinder actuators, etc. From Eq.
\[ H \]
we saw that the contact pressure is formulated based on the
coordinate system CS1 depicted again in Fig. 10 where the origin
at the centerline point meets the condition \((r_c = \sigma_c)\).
If a new coordinate system CS2 (Fig. 10) is established at the
free end of the tube, then point P1 in section I has two sets of
coordinate \((z_1, r)\) and \((z, r)\) for CS1 and CS2, respectively, while
point P2 in the section II has two sets of coordinate \((z_2, r)\) and
\((z, r)\) for CS1 and CS2, respectively. By the geometrical relation,
Eq. (31) can be obtained. Combining Eqs. (19) and (20) lead to
Eq. (32).

\[
\begin{align*}
Z &= Z_1 + L_2 \\
L_2 &= Z_2 + Z \Rightarrow \left\{ \\
Z_1 &= Z - L_2 \\
Z_2 &= L_2 - Z
\right. (31)
\end{align*}
\]

\[
\sigma_r = -p_z \exp \left( -\frac{\mu z_1}{(2\alpha - 1) t_0} \right), \quad 0 \leq z_1 \leq l_1
\]

\[
\sigma_r = -p_z \exp \left( -\frac{\mu z_2}{(2\alpha - 1) t_0} \right), \quad 0 \leq z_2 \leq l_2 (32)
\]

Substituting Eq. (31) into Eq. (32), Eq. (33) is obtained, which can
be simplified to Eq. (34). By substituting Eq. (25) into Eq. (34),
the contact pressure in the form given in Eq. (35) is obtained.

\[
\sigma_r = -p_z \exp \left( \frac{\mu (Z - L_2)}{(2\alpha - 1) t_0} \right), \quad L_2 \leq z \leq L
\]
Thus, the ratio, (33)

\[ \frac{\sigma_r}{\sigma_i} = -p \exp\left( \frac{\mu (Z - L_2)}{(2\alpha - 1)\mu} \right), \quad 0 \leq z \leq L_2 \]

Equation (35) can be used to plot a friction hill envelope for the guiding zone. Figure 11 shows an example of a friction hill envelope for a guiding zone with 200 mm tube length, 50 mm tube diameter, and 2 mm wall thickness at an internal pressure \( P_i \). As seen in Fig. 11, when \( \mu = 0.2 \), the maximum pressure acting on the tube-die interface is over two times higher than the fluid pressure.

A more generic form, however, is a dimensionless scheme that shows the variation in maximum direct contact pressure to fluid pressure ratio (\( \sigma_r / P_r \)) with tube length to diameter ratio \( (L_o / D_o) \) for various friction conditions. The variable \( R_p \) is defined as ratio of contact pressure, \( \sigma_r \), to internal pressure, \( P_r \). By considering the tube as a thin wall structure where \( \alpha > 1 \), the ratio \( R_p \) can be expressed as a function of interface friction and geometric variables. From Eq. (36), it can be seen that the maximum \( R_p \) occurs at the loaded end of the tube where \( Z = L_o \), as expressed in Eq. (37).

\[ R_p = \frac{\sigma_r}{P_r} = \left( \frac{2\alpha - 2}{2\alpha - 1} \right) \exp\left( \frac{\mu z}{(2\alpha - 1)\mu} \right) = \exp\left( \frac{\mu z}{D_o} \right) \]

where

\[ \left( \frac{2\alpha - 2}{2\alpha - 1} \right) = 1, \quad (2\alpha - 1)\mu = \left( \frac{2\mu z}{D_o} - 1 \right) = \frac{1}{D_o} \]

Equation (37) shows that the maximum contact pressure along the tube length depends on the friction coefficient \( \mu \) and the ratio of tube length \( L_o \) to the tube outer diameter \( D_o \). Figure 12 shows the variation in \( R_p \) with the \( L_o / D_o \) ratio. It can be seen that maximum contact pressure increases rapidly with the increase in \( L_o / D_o \) ratio. Thus, the ratio, \( L_o / D_o \), is a critical parameter in the design of the guiding zone for tube hydroforming systems.

The friction hill envelope given in Fig. 11 and variation in \( \text{Max} R_p \) versus \( L_o / D_o \) given in Fig. 12 have been established based on Eq. (35), which is valid for low strain range. Figures 11 and 12 can therefore be applied when the tube length to diameter ratio \( L_o / D_o \) does not exceed 4 and a maximum coefficient of friction of \( \mu = 0.2 \). Appendix B gives details on derivation of expressions for determining state variables at higher strain level (\( e_z = 0.1 - 2.0 \)). An example of tube-die contact pressure distribution at higher strain level is given in Fig. 13. These curves were generated by the derived expression given in Appendix B. Figure 13 shows the maximum pressure that can be generated when an internal pressure of 60 MPa is applied on a tubular material with a strain hardening exponent, \( n = 0.3 \), and a strength coefficient, \( K = 500 \) MPa.

As can be seen from Fig. 13, an original tube length of 500 mm \( (L_o / D_o = 10) \) result in a contact pressure of up to 450 MPa when a coefficient of friction of \( \mu = 0.4 \) is exhibited at the interface. This pressure is over seven times higher than the forming pressure. The figure also shows that the increase in interface pressure is highly influenced by the increase in \( L_o / D_o \) ratio. Figure 13 also shows the comparison between the analytical model and FEA for tube lengths of 200 mm and 400 mm. With 200 mm tubing, FEA shows contact pressure at the tube-die interface of 86 MPa, 113 MPa, and 138 MPa for \( \mu = 0.1 \), \( \mu = 0.2 \), and \( \mu = 0.3 \), respectively, whereas the analytical model exhibited contact pressures of 87 MPa, 109 MPa, and 133 MPa for \( \mu = 0.1 \), \( \mu = 0.2 \), and \( \mu = 0.3 \) respectively. As the tube is pushed in the guiding zone, tube thickening will occur and the tube length will be shortened. For an initial tube length of 200 mm, FEA shows that final deformed tube lengths were 181 mm, 147 mm, and 127 mm for \( \mu = 0.1 \), \( \mu = 0.2 \), and \( \mu = 0.3 \), respectively. The analytical model resulted in final tube lengths of 181 mm, 150 mm, and 130 mm for \( \mu = 0.1 \), \( \mu = 0.2 \), and \( \mu = 0.3 \), respectively. Good agreement can be observed between analytical model and FEA results.
6 Conclusions

Closed-form solutions that characterize the guiding zone tribotest for tube hydroforming were established based on a mechanistic approach. From the derived analytical model field, variables can be computed along the tube. These variables include (a) contact pressure distribution, (b) effective stress and strain distribution, (c) longitudinal stress and strain distribution, and (d) hoop stress distribution.

Based on the derived equation for the contact pressure at the tool-tube interface, an expression for determining the coefficient of friction for the guiding zone tribotest was established. This expression is a function of tube geometric variables, internal pressure, and friction force obtained from tribotest experiment.

Through this study, friction hill envelopes for the guiding zone were established. These friction hill envelopes show the variation in contact pressure along the tube length as a function of friction coefficient. The study has also shown that the maximum contact pressure at the tool-tube interface increase rapidly with the increase in $L_0/D_0$ ratio. Thus, the ratio $L_0/D_0$ is a critical parameter in the design of the guiding zone for tube hydroforming systems.

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Nomenclature

- $\mu$ = friction coefficient
- $F_1$ = friction force
- $F_r$ = normal load
- $F_1,F_2,F_3$ = tube end load
- $P_i$ = internal pressure
- $d$ = inner diameter
- $D_o$ = outer diameter
- $r_i$ = inner radius
- $r_o$ = outer radius
- $L$ = deformed tube length
- $L_0$ = initial tube length
- $l_i$ = length of section I
- $l_z$ = length of section II
- $t$ = deformed tube thickness
- $t_0$ = original tube thickness
- $V$ = volume of deformed tube
- $V_0$ = volume of initial tube
- $\alpha$ = ratio of tuber outer radius to initial tube wall thickness
- $D_i$ = internal diameter of the tube
- $\lambda$ = ratio of tuber outer radius to instantaneous tube wall thickness
- $R_p$ = ratio of contact pressure to internal pressure
- $\bar{\sigma}$ = effective stress
- $\bar{e}$ = effective strain
- $K$ = strength coefficient
- $n$ = strain hardening exponent
- $\sigma_r$ = longitudinal stress
- $\sigma_t$ = radial stress
- $\sigma_\theta$ = hoop stress
- $\varepsilon_r$ = longitudinal strain
- $\varepsilon_t$ = radial strain
- $\varepsilon_\theta$ = hoop strain
- $z, Z_1, Z_2$ = longitudinal coordinate of the tube

Appendix A: Derivation of Strain $\varepsilon_z$ in Section II of the Tube in the Guiding Zone

The longitudinal strain $\varepsilon_z$ of section II can be derived from Eqs. (4), (5), (7), and (10). We set the tube length $l_z$ and the original point on the z-axis to where $\sigma_z = \sigma_r$ (refer Fig. 5). Because section II is near the free tube end, the longitudinal strain $\varepsilon_z$ is small. Thus the term $dt$ in Eq. (6) can be neglected and Eq. (6) can be reduced to Eq. (A1). The radial stress and longitudinal stress given in Eqs. (A2) and (A3) are obtained by combining Eqs. (4), (5), and (10). From Eq. (A3), Eq. (A4) can be derived. Substituting Eqs. (A1) and (A2) into Eq. (A4) yields Eq. (A5).

$$\frac{d\sigma_z}{dz} = -\frac{\mu}{t} \sigma_r$$  \hspace{1cm} (A1)

$$\sigma_z = -\frac{2r_o}{t} K \beta^{n+1} \varepsilon_z^n$$  \hspace{1cm} (A2)

$$\sigma_z = -\frac{2r_o}{t} K \beta^{n+1} \varepsilon_z^n$$  \hspace{1cm} (A3)

$$\frac{d\sigma_r}{dz} = -\frac{1}{t} K \beta^{n+1} n \varepsilon_z \frac{1}{dz}$$  \hspace{1cm} (A4)

$$\left(\frac{r_o}{t} - \frac{1}{2} K \beta^{n+1} n \varepsilon_z \frac{1}{dz}\right) = \frac{\mu r_i}{t} P_i - \frac{\mu}{2t} K \beta^{n+1} \varepsilon_z^n$$  \hspace{1cm} (A5)

Section II is near the free tube end and the axial load from the punch drops significantly in this section, which results in small axial strain $\varepsilon_z$. Thus, it is reasonable to assume that the deformed tube thickness $t$ approximate original length $t_0$. Let $r_i/t_0 = \alpha$, $r_o/t_0 = \alpha - 1$, and substitute into Eq. (A5) to obtain Eq. (A6). Integrating Eq. (A6), and invoking the boundary conditions that $\varepsilon_z = 0$ at $z = 0$, an explicit expression for $\varepsilon_z$ can be obtained as given in Eq. (A7). By substituting Eq. (A7) into Eqs. (A2) and (A3), the radial and longitudinal stress can be obtained as functions of interface friction, internal pressure, wall thickness, and the ratio of tube outer radius to tube wall thickness.

$$\left(\alpha - \frac{1}{2} K \beta^{n+1} n \varepsilon_z \frac{1}{dz}\right) = \frac{\mu r_i}{t} P_i - \frac{\mu}{2t} K \beta^{n+1} \varepsilon_z^n$$  \hspace{1cm} (A6)

$$\varepsilon_z = \frac{2(\alpha - 1)}{K \beta^{n+1} P_i (1 - \exp(-\frac{\mu z}{(2\alpha-1)t}))}$$  \hspace{1cm} (A7)

Appendix B: Explicit formula for Evaluation of State Variables at High Strain Range

Figure 14 shows a typical characteristic curve (Z versus $\varepsilon_z$) obtained from Eq. (16). Figure 14 shows that longitudinal strain $\varepsilon_z$ varies approximately linearly with longitudinal tube length $z$ at high strain range of ($\varepsilon_z = 0.1 - \varepsilon_z = -2.0$). It is therefore possible to get an explicit formula for evaluating the state variables such as contact pressure at high strain level. From the linear relationship, Eq. (16) can be simplified to Eq. (B1).

$$z = z_{0.1} + \text{Slp} (\varepsilon_z + 0.1)$$  \hspace{1cm} (B1)

where “Slp” is the slope of the curve and $Z_{0.1}$ is the longitudinal length where the longitudinal strain is $\varepsilon_z = 0.1$. The slope of the curve can be computed approximately by substituting $\varepsilon_z = -1.0$ into the integrand at the left side of Eq. (16) which leads to Eq. (B2). $Z_{0.1}$ can be obtained by substituting $\varepsilon_z = 0.1$ into Eq. (18), as shown in Eq. (B3). Note that the slope, Slp, is a function of $K$, $n$, $r_i$, $t_0$, $P_i$, and $\mu$.  

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Substituting Eqs. (B2) and (B3) into Eq. (B1), an explicit expression for the longitudinal strain can be obtained, Eq. (B4).

\[
\sigma_z = -\frac{K\beta^{n+1}(0.1 + b\mu(z-c))t_0e^{(0.1 + b\mu(z-c))}}{2(r_a - t_0e^{0.1 + b\mu(z-c)})} + P_12(t_a - t_0e^{0.1 + b\mu(z-c)})
\]

(B6)

Appendix C: Integration of Equation (17)

\[
\int az = \int \left(\frac{a}{2} - \frac{1}{2}\right)K^{n+1}n(-e_z)^{n-1}de_z
\]

(C1)

Let \( a = \left(\frac{a}{2} - \frac{1}{2}\right)K^{n+1} \), \( b = \frac{\mu}{2t}K^{n+1} \), \( c = \frac{\mu(a-1)}{t}p_i \)

(C2)

\[
\int az = \int \frac{a(t(e_z)^n - e_z)}{b(e_z)^n + c} + \frac{a}{b} \int \frac{b(e_z)^n + c}{b(e_z)^n + c} + c_0 = z
\]

(C4)

The boundary conditions are such that when \( z=0 \), \( \sigma_z = \sigma_r \). By substituting \( e_z = 0 \) at \( z=0 \) into Eq. (C4), the constant \( c_0 \) can be determined.

References


