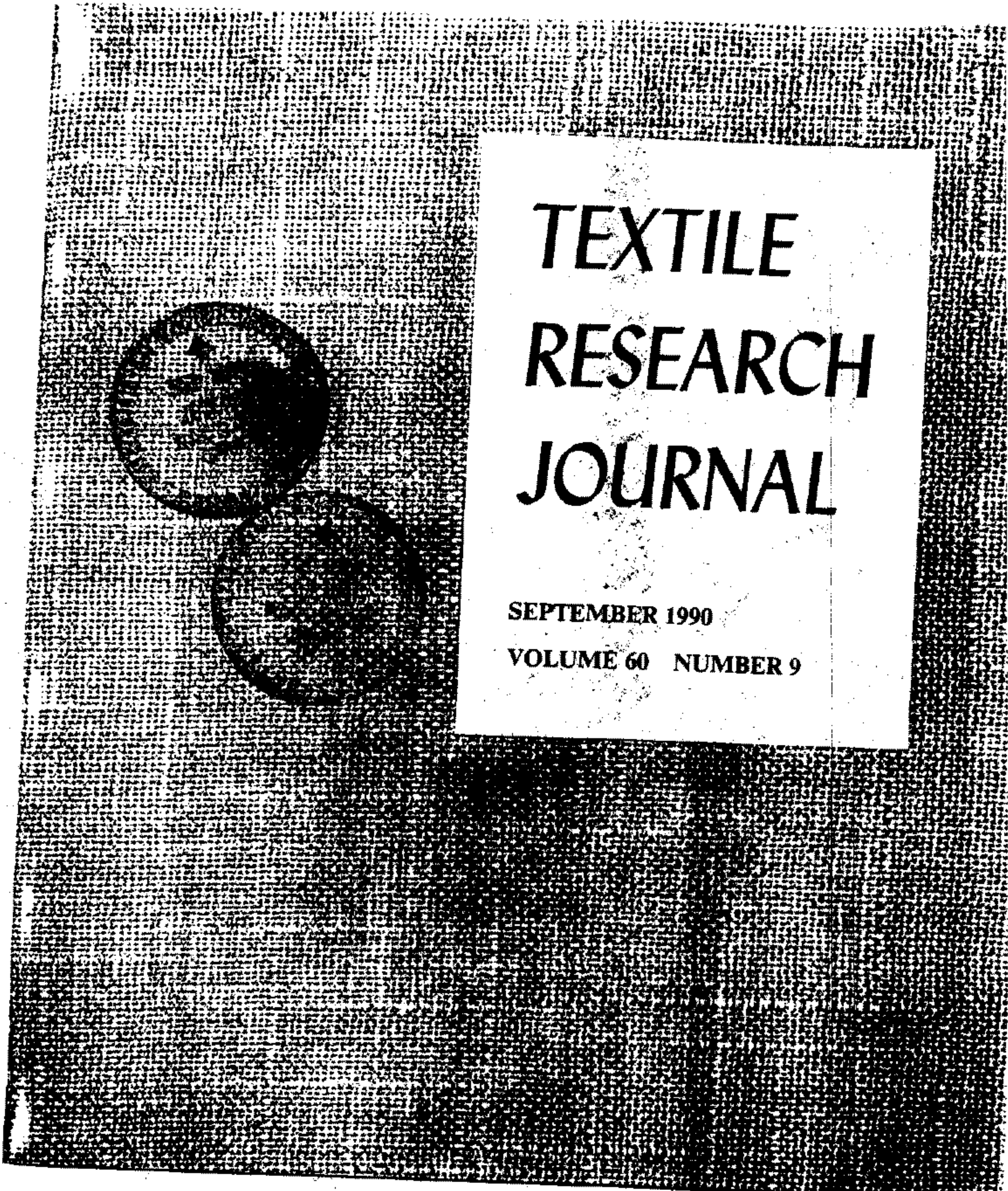

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tionship was used to approximate the nonlinear bending stiffness in fabrics. The advantage of this method is that the fabric nonlinear bending behavior, which is inherent in most fabrics, can be well represented; this property may not always be obtained from the traditional cantilever beam test.

Understanding the bending behavior of fabrics is an important topic in scientific textile research as well as in industrial applications. Fabric bending behavior has been the focus of many investigations. Most studies either concern theoretical modeling of the fabric bending properties or actual measurement using various methods.

Peirce first introduced a linear model of bending behavior of fabrics in the 1930s [9]; this is probably the most simple and useful model that is still widely applied in many textile areas. By using a first-order approximation, this model is based on the assumption that the bending rigidity is independent of the curvature. Most fabrics display a nonlinear moment-curvature relationship during bending. Strictly speaking, the linear bending behavior model usually cannot satisfactorily describe the fabric bending process. To address this inadequacy, several nonlinear bending models have been proposed in the literature. They include Grosberg's frictional couple theory [5], which we later developed by adding the effect of fabric weight [2]; the bilinear bending theory was proposed and developed successively by Huang [6], Leaf [8], and Ghosh [3]. These nonlinear bending models can more closely describe the actual moment-curvature relations of fabrics, and thus are preferable whenever a theoretical investigation of fabric behavior in the bending or buckling processes becomes necessary.

There are a variety of methods or standards of measuring woven fabric bending rigidity. Two of these have been widely adopted by scientific research institutions and textile industries: the ASTM method D1388 [1] (the cantilever beam test) and the Kawabata pure bending test [7].

In the ASTM method, a critical "bending length" of a beam is measured when the secant line across both points of the beam's fixed and free ends makes an angle of 41.5° to the horizontal. The bending rigidity B of the beam is defined as

$$B = w\left(\frac{L}{2}\right)^3 \quad , \tag{1}$$

where w is the fabric weight per unit area and L is the bending length. This method was developed based on the linear beam theory [10]. Actually, most fabrics possess a strong nonlinear bending behavior, as described by Grosberg [4]. This method is only capable of providing an average value of the bending rigidity of the fabric, an obvious shortcoming.

The Kawabata bending tester is designed to directly measure the fabric moment-curvature relationship in pure bending. By subjecting a fabric sample to a

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monotonically increasing curvature, the corresponding moment acting on the fabric is recorded. With this technique, the actual nonlinear bending behaviors of fabrics can be measured, but the resolution of the moment-curvature relationship is difficult for small curvatures, precisely where most of the nonlinear behavior occurs.

In contrast to the two methods discussed above, our paper proposes a new method of indirectly measuring fabric bending behavior using the ASTM cantilever beam test as the basis. During the cantilever beam test, the x–y coordinates of the fabric bending curve are collected. With this information, the actual fabric moment-curvature relationship can be derived using techniques of numerical regression and differentiation. The analytical background, together with numerical and experimental validation of the proposed method, is described in the following section.

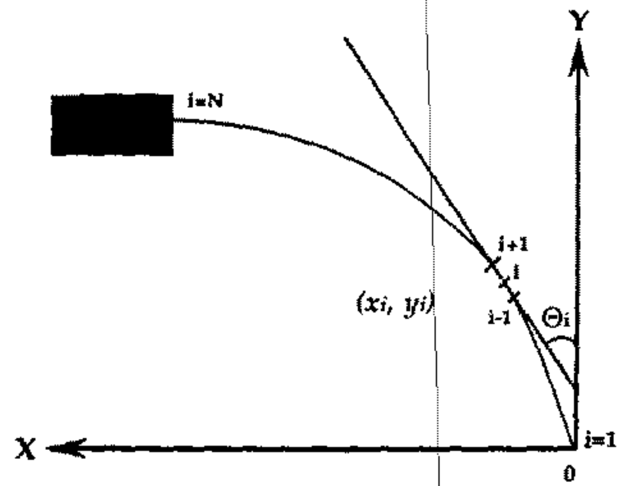

FIGURE 1. The geometry of a cantilever beam.

Indirect Measurement of Moment-Curvature Data

The new method first requires accurate collection of the x and y coordinates along the fabric bending curve. Numerical analysis techniques are then applied to these data to calculate the values of moment and curvature along the fabric length, from the free end point to the fixed end point. Since curvature is a function of the second-order derivatives of fabric displacement, the errors associated with numerical differentiation normally would be very large. Thus, choosing a proper numerical procedure in order to control and reduce the errors involved in the numerical algorithm is crucial to the success of the method.

The geometry of the cantilever beam is shown in Figure 1. The origin of the Cartesian coordinate system is fixed at the free end of the deformed fabric sample, where both the bending moment and curvature are zero. It is not necessary to choose the fabric length according to the value of bending length, as required by ASTM method D1388 [1]. In that method, it is advantageous to choose a fabric sample with a longer length to induce higher curvature in the fabric sample.

The width of the fabric sample normally can be from 2.5 cm to 8 cm. Generally, the samples must not be creased initially or possess initial curvature. It is possible that the free end of a sample may twist somewhat during tests. If this occurs, we recommend that the coordinates of the fabric bending curve be taken along the center line of the sample.

A very important premise to the success of this method is the precise readings of the series of coordinates along the fabric bending curve. This will greatly reduce the initial errors in the input data, thus allowing the computation algorithm to successfully predict the curvature.

The following description gives the methodology for computing moment-curvature (M-K) data from a deformed fabric sample. This procedure is organized into three steps.

1. Use of a series of coordinates (x_i, y_i) *to construct the series of polar coordinates* (S_i, θ_i): The continuous curve of the cantilever beam is now recorded as a series of discrete Cartesian coordinate pairs (x_i, y_i) as shown in Figure 1. The total number of data pairs is N. It is obvious from Figure 1 that $x_1 = 0$ and $y_1 = 0$. At any point i $(1 \le i \le N)$ on the curve, the arc-length parameter s_i and differential increment of the length Δs_i may be defined as

$$\Delta s_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad , \qquad i = 1, 2, \ldots, N - 1 \quad , \tag{2}$$

and

$$s_{i+1} = s_i + \Delta s_i \quad , \quad i = 1, 2, \ldots, N - 1 \quad , \tag{3}$$

where $s_1 = 0$.

In order to smooth the measured displacement data and reduce the initial measurement errors associated with these data, we use a least squares regression technique. Using the displacement coordinates (x_i, y_i) as input, we selected a fifth-order polynomial to simulate the relationship $y = f(x)$. Since the deflection of a uniformly loaded beam with a linear bending stiffness is expressed by a fourth-order polynomial function [11], we chose a fifth-order polynomial to incorporate the nonlinear bending stiffness present in most fabrics.

Thus the angle of the tangent at the point i, θ_i, is computed as

$$\theta_i = \frac{\pi}{2} - \tan^{-1}\left(\frac{df}{dx}\right)\bigg|_{x=x_i} \quad ,$$

$$i = 1, 2, \ldots, N-1 \quad . \quad (4)$$

At the end point N, a fixed end boundary condition is imposed so that $\theta_N = \pi/2$. With this formulation, we obtained a series of polar coordinate data pairs (s_i, θ_i).

2. Calculation of the moments M_i: In a cantilever beam test, the internal bending moment at a cross section located at an arbitrary point s is caused by the fabric's own weight. Analytically, it can be expressed as [10]

$$M = \int_0^s ws' \sin\theta ds' \quad , \qquad (5)$$

where w is the fabric weight per unit area and s' is a dummy variable.

For our discretization scheme, the internal moment at a point $i+1$, M_{i+1}, can be written as

$$\begin{aligned} M_{i+1} &= \int_0^{s_{i+1}} ws' \sin\theta ds' \\ &= \int_0^{s_i} ws' \sin\theta ds' + \int_{s_i}^{s_{i+1}} ws' \sin\theta ds' \\ &= M_i + \Delta M_i \quad , \quad i = 1, 2, \ldots, N-1 \quad , \end{aligned} \quad (6)$$

where ΔM_i is the differential increment of moment between point i and $i+1$, $M_1 = 0$, and ΔM_i can be approximated as

$$\begin{aligned} \Delta M_i &= w \sin\bar{\theta}_i \int_{s_i}^{s_{i+1}} s'ds' \\ &= w \sin\bar{\theta}_i \left(s_{i+1} - \frac{\Delta s_i}{2}\right)\Delta s_i \quad , \end{aligned}$$

$$i = 1, 2, \ldots, N-1 \quad , \quad (7)$$

where

$$\sin(\bar{\theta}_i) = \sin\left(\frac{\theta_{i+1} + \theta_i}{2}\right) \quad .$$

3. Calculation of the curvatures $K_i = d\theta_i/ds$: To obtain the local curvature $K_i = d\theta_i/ds$, we again used the method of least squares fifth-order polynomial regression to simulate the relation of $\theta = F(s)$.

In this study, the inclining angle θ is a monotonically increasing and slow varying function of the arc length s. We found from computational practice that there was little difference using third-, fourth-, or fifth-order polynomials to simulate the relationship of $\theta = F(s)$. We selected a fifth-order to reduce the complexity of the computer program, since we had already incorporated this regression analysis subroutine in the program.

The purpose of this regression procedure is to smooth the data and to avoid possible large numerical errors, which may be produced either by using the finite difference method or the second-order derivatives when the values of curvature K are to be generated. By differentiating the regression function $F(s)$, the local curvatures K_i are obtained from

$$K_i = \frac{dF}{ds}\bigg|_{s=s_i} \quad , \quad i = 1, 2, \ldots, N \quad . \qquad (8)$$

At the conclusion of step 3, we computed the M-K coordinates (K_i, M_i) for one fabric sample. The M-K data can then be used to approximate the nonlinear bending stiffness.

Approximation of Bending Stiffness

Bending stiffness $B(K)$ is defined as the change in moment divided by the change in curvature or dM/dK. Since most fabrics display nonlinear bending behavior, bending stiffness is a function of curvature.

Nonlinear bending stiffness can often be approximated using a bilinear fit [3, 6, 8]. In bilinear theory, the moment versus curvature relationship is continuous, but the bending stiffness versus curvature is discontinuous. Bending stiffness, although nonlinear, is a continuous function of curvature in the fabrics. In this study, we adopted a similar consideration in the methodology for computing $B(K)$ using indirectly measured M-K data. But we used a method of quadratic-linear fit to guarantee the continuity of moment and bending stiffness over the full range of measured curvatures.

For practical considerations, when the bending stiffness of a specific fabric needs to be quantified, at least several representative samples should be measured. The average moment-curvature relationship from those samples should characterize fabric. Moment-curvature data are computed from several fabric samples. A least squares regression method is used to obtain the "mean value" of the M-K curve. For all measured data, a quadratic regression and a linear regression are taken. The second-order polynomial is merely a best-fit approximation based on the shape of experimental M-K curves. This model more closely simulates the bending stiffness as a continuous function of the curvature than a discontinuous, bilinear fit. The resultant M-K curve can be written as

$$M_1 = a_1 + b_1 K + c_1 K^2 \qquad (9)$$

and

$$M_2 = a_2 + b_2K \quad , \tag{10}$$

where a_1 and a_2 are not necessarily equal to zero.

The value of B obtained by differentiating Equation 9 will be used for small values of K. Values of B in this range may be termed the "initial bending stiffness." Obviously, B now is a linear function of the curvature K. The value of B obtained by differentiating Equation 10 is to be used for large values of K, the result being called the "ultimate bending stiffness." Equations 9 and 10 are not the final forms of M-K curve to be constructed in this study. The moment-curvature relationship of a fabric should meet the conditions of causality ($M = 0$ for $K = 0$), continuity, and uniqueness. To build an M-K curve to satisfy these requirements, we use the following method.

Considering the fact that B is a continuous function for all values of K, the following condition of continuity must be satisfied:

$$\frac{dM_1}{dK} = \frac{dM_2}{dK} \quad \text{at} \quad K = K_0 \quad . \tag{11}$$

Solving Equation 11 yields

$$K_0 = \frac{b_2 - b_1}{2c_1} \quad . \tag{12}$$

Integrating the functions of dM_1/dK and dM_2/dK, using the conditions of $M = 0$ for $K = 0$ and $M_1 = M_2$ at $K = K_0$, the new M-K curve is given by

$$M_1 = b_1K + c_1K^2 \quad , \quad |K| \leq K_0 \quad ,$$

and

$$M_2 = M_0 + b_2K \quad , \quad |K| > K_0 \quad , \tag{13}$$

where $M_0 = (b_1 - b_2)K_0 + c_1K_0^2$.

Our regression method computes a continuous M-K curve that is more representative of a real fabric moment-curvature relationship than a bilinear curve.

Numerical Verification

We wrote a Fortran computer program based on the algorithm described above. In order to validate the algorithm, we used a typical example of linear bending to check the performance of the program.

The nondimensional equation governing an elastica in large deformation may be written as [2]

$$\frac{d^2\theta}{d\bar{s}^2} = \bar{w}\bar{s} \sin \theta \quad , \tag{14}$$

where $\bar{s}$ is the normalized length variable starting from the free end, $\bar{s} = s/L$, $0 < \bar{s} < 1$. The symbol $\bar{w}$ denotes the weight factor, where $\bar{w} = \frac{wL^3}{B}$. For a cantilever beam, the boundary conditions are $d\theta/d\bar{s} = 0$ at $\bar{s} = 0$ and $\theta = 0$ at $\bar{s} = 1$. When the value of $\bar{w}$ is known, a unique solution of θ at the free end can be found using a numerical integration method. Thus the fabric bending curve expressed as the x–y coordinates is also determined. In this study, we used the fifth-order Runge-Kutta method to solve the initial value problem in Equation 14; the implementation of this method appears in an earlier paper [2]. Note that in the process of numerical evaluation, the normalized curvature ($\bar{K}$) is always equal to the normalized moment $\bar{M}$, since

$$\bar{K} = \frac{d\theta}{d\bar{s}} = \int_0^{\bar{s}} \bar{w}\bar{s}' \sin \theta d\bar{s}' = \bar{M} \quad . \tag{15}$$

We solved Equation 14 for a given value of $\bar{w}$, and also calculated 21 x–y coordinates along the deformed fabric sample from $\bar{s} = 0$ to $\bar{s} = 1$ ($\Delta\bar{s} = 0.05$), along with the value of $d\theta/d\bar{s}$ (equivalent to $\bar{K}$ and $\bar{M}$). On the other hand, our program uses the obtained displacement data, in x–y pairs, and the value of $\bar{w}$ as input to calculate the values of $\bar{K}$ and $\bar{M}$. We can then compare the values of $\bar{K}$ and $\bar{M}$ calculated using these two methods for validation.

Two typical values of $\bar{w}$, 10 and 40, were taken in the numerical evaluation. Using the Runge-Kutta method and solving Equation 14, the values of the normalized curvature at the fixed end ($\bar{s} = 1$) were 3.746 for $\bar{w} = 10$ and 8.459 for $\bar{w} = 40$, respectively.

The numerical results of using these two programs are shown in Figure 2 for $\bar{w} = 10$ and Figure 3 for $\bar{w} = 40$. For $\bar{w} = 10$, we limited the accuracy of the input

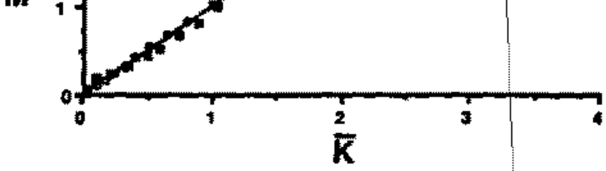

FIGURE 2. Comparison of the theoretical predictions and the results of our method, $\bar{W} = 10$.

data (x_i, y_i) to two digits after the decimal point, corresponding to a maximum absolute error of ±0.005, and to three digits after the decimal point, corresponding to a maximum absolute error of ±0.0005, respectively. As shown in Figure 2, the results of these two programs are matched quite satisfactorily, considering the fact that we applied two approximations of the fabric curve in our method to achieve the moment-curvature relationship. The figure also shows that the performance of our method depends on the accuracy of the input data. As the precision of the input data increases, the better the results.

Figure 3 displays the case of $\bar{w} = 40$. The maximum absolute errors of the input data were set for three levels: ±0.005, ±0.0025, and ±0.0005. The numerical results show that deviation from the exact solutions is relatively large for the input data with an error level of ±0.005. But the results are good for the other two cases where the input data have error levels of ±0.0025 and ±0.0005, respectively. We can conclude from these two test examples that, ideally, our method can very closely recover the actual moment-curvature relationship of fabrics with great accuracy. However, the accuracy of the input data is key to the success of the method. In order to assure convergence, the fabric bending curve x–y coordinates must be measured as accurately as possible.

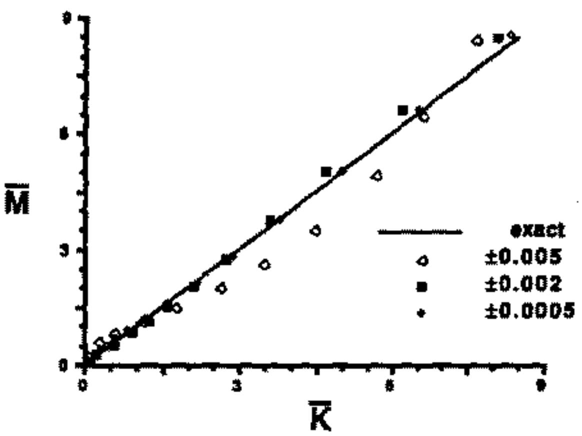

FIGURE 3. Comparison of the theoretical predictions and the results of our method, $\bar{W} = 40$.

Experimental Verification

We conducted a cantilever beam test using three different fabrics (A, B, C) and a cellulose film (D). A description of these materials is presented in Table I. The values of bending stiffness were measured with the KES-FB2 tester for comparison purposes.

TABLE I. Physical properties of fabric samples.

Fabric[a]	Direction, warp or fill	Averaged length, cm	Unit weight, mgf/cm^2	KES data bending rigidity, $gf \cdot cm^2/cm$
A	fill	5.6	6.3	0.022 (9.8)[b]
B	warp	7.4	9.1	0.1212 (15.7)
C	warp	17.6	25.2	2.215 (27.8)
D	warp	13.6	13.6	3.293 (2.96)

[a] A, 106 ends/inch, 88 picks/inch, polyester/cotton, plain weave; B, 81 ends/inch, 52 picks/inch, polyester/cotton, plain weave; C, 52 ends/inch, 40 picks/inch, 100% Nomex, plain weave; D, polyester film. [b] CV% in parentheses, five replications.

A fixture was used to hold the fabric samples and to simulate the fixed end or cantilever boundary condition as seen in Figure 4. The surface of the fixture was kept on a horizontal plane. The fabric bending shapes were displayed using an IRI image processing vision system; the picture of the fabric curve was digitized by the system. The pixel coordinates of the fabric were recorded and converted into x–y coordinates.

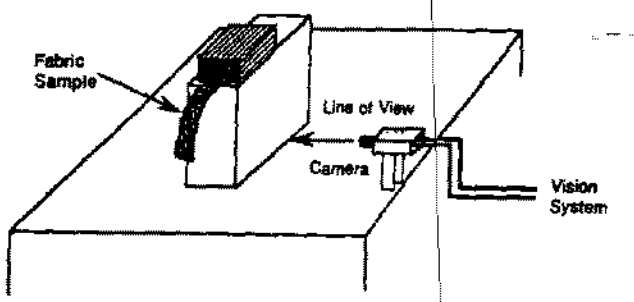

FIGURE 4. Experimental apparatus of the cantilever beam test—overview.

We used the measured data of fabric coordinates as the input of the program to compute the moment-curvature relationship of the fabric samples. We found that at the two extremes of curvature, the degree of accuracy of the data was normally relatively low. The curvature is small near the free end of the fabric; in this region, a small value of absolute error in the measurements may cause a large relative error in displacement, so it is easy to produce distorted data in the region of small curvatures. On the other hand, the regression technique generates some deviations near each end of the curve from the exact solution due to lack of data points. For these reasons, we do not normally use the data located at two ends of the predicted M-K data.

The numerical results are plotted in Figures 5, 6, 7, and 8 for the four samples A, B, C, and D, respectively. The M-K curves simulated using the regression method also appear in these figures. The analytical expressions for these simulated M-K curves and the bending rigidity are listed in Table II.

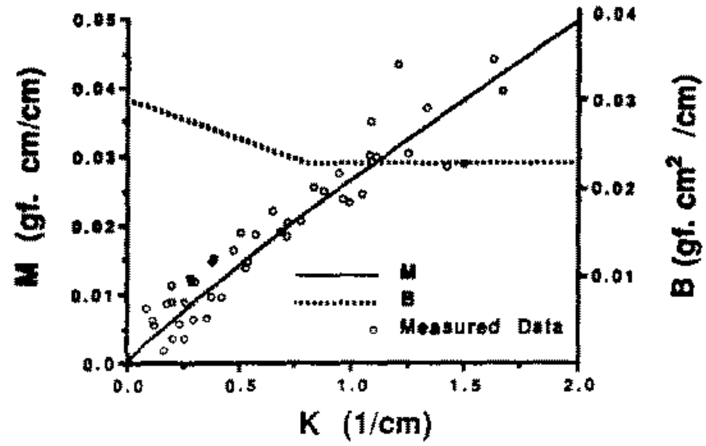

FIGURE 5. Moment-curvature relationship constructed using our method, fabric A.

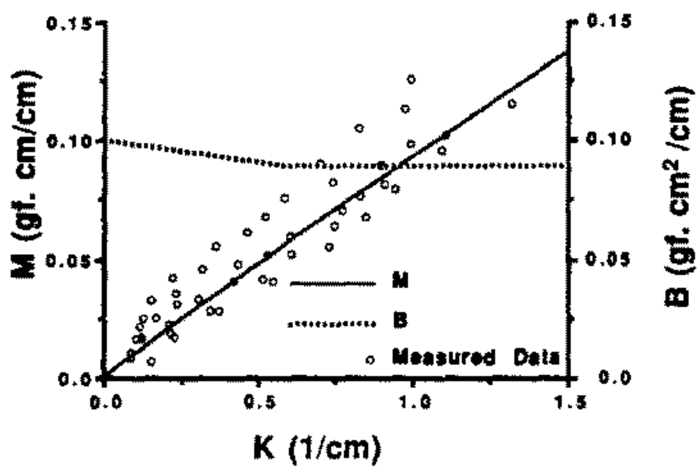

FIGURE 6. Moment-curvature relationship constructed using our method, fabric B.

TABLE II. Numerically constructed fabric M-K curve and bending stiffness.

Fabric	Moment M (M = gf·cm/cm, K = 1/cm)		Bending stiffness B (B = gf·cm²/cm, K = 1/cm)	
A	$0.0306\|K\| - 0.0046K^2$,	$\|K\| \le 0.801$	$0.0306 - 0.0092\|K\|$,	$\|K\| \le 0.801$
	$0.0233\|K\| + 0.0029$,	$\|K\| > 0.801$	0.0233	$\|K\| > 0.801$
B	$0.1\|K\| - 0.0092K^2$,	$\|K\| \le 0.587$	$0.1 - 0.0184\|K\|$,	$\|K\| \le 0.587$
	$0.0892\|K\| + 0.0032$,	$\|K\| > 0.587$	0.0892	$\|K\| > 0.587$
C	$5.2043\|K\| - 4.4466K^2$,	$\|K\| \le 0.3028$	$5.2043 - 8.8932\|K\|$,	$\|K\| \le 0.3028$
	$2.511\|K\| + 0.4078$,	$\|K\| > 0.3028$	2.511	$\|K\| > 0.3028$
D	$3.5352\|K\| - 2.2192K^2$,	$\|K\| \le 0.1246$	$3.5352 - 4.4384\|K\|$,	$\|K\| \le 0.1246$
	$2.9823\|K\| + 0.0344$,	$\|K\| > 0.1246$	2.9823	$\|K\| > 0.1246$

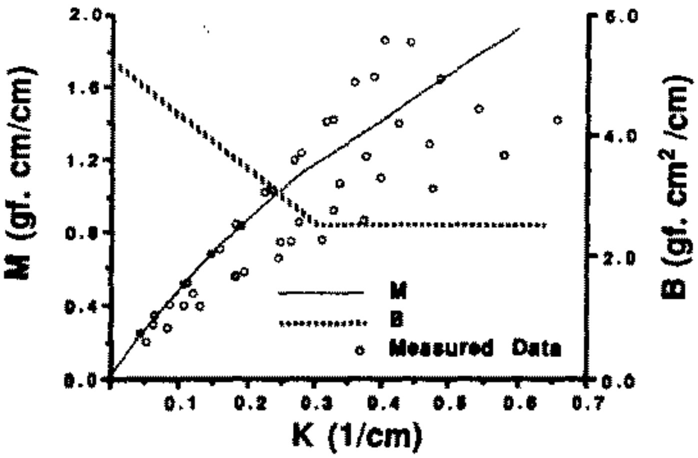

FIGURE 7. Moment-curvature relationship constructed using our method, fabric C.

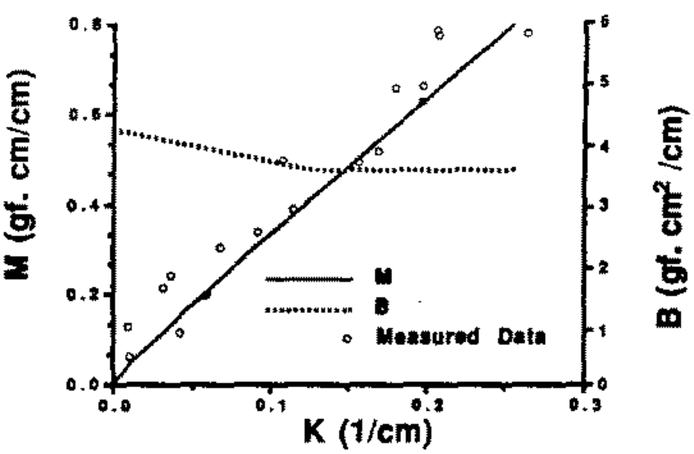

FIGURE 8. Moment-curvature relationship constructed using our method, cellulose film (D).

From these figures, we see that the numerically constructed M-K curves using our method in general fit the measured data. The important feature of the nonlinear phenomenon observed from most fabric bending is clearly demonstrated: the value of initial bending stiffness is greater than that of ultimate bending stiffness, and transition from the former to the latter is smooth and continuous. The ability to show actual bending properties of real fabrics certainly is one advantage of our method.

As shown in Figure 7, the measured data of moment-curvature of fabric C displayed a relatively large degree of divergence. This was not caused by the numerical errors involved in the method, but by the fabric's own variable characteristics in bending. Table I shows that from the Kawabata measurement, the averaged value of bending rigidity of fabric C is 2.215 $gf \cdot cm^2/cm$, but the value of correlation variance about that data is as high as 27.8%. That is an indication of the large degree of variation of the fabric bending properties.

In order to validate the numerically constructed M-K curves of these four materials, it is necessary to use this information together with some other data (the values of sample length and weight per unit area) to compute the deformed fabric shapes and compare them with the experimental shapes.

The measured and computed bending curves of cantilever beams are plotted in Figures 9, 10, 11, and 12 for the samples A, B, C, and D, respectively. For fabrics A, B, and C, the calculated curves using our method are located around the middle range of measured fabric curves, indicating good performance of the numerical algorithm. For the cellulose film (D), the bending curve calculated using our method matches

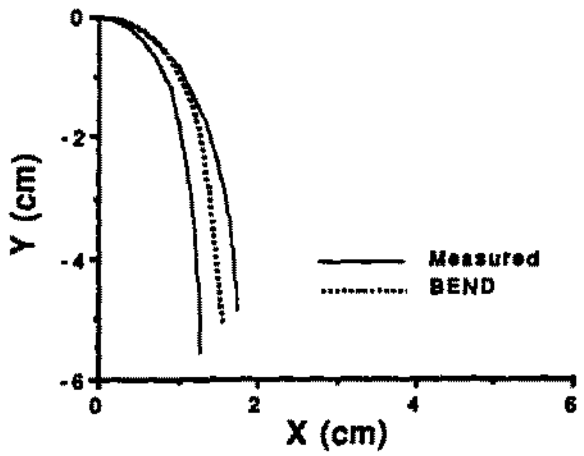

FIGURE 9. Comparison of bending shapes obtained by experimental observation using KES data and our method, fabric A.

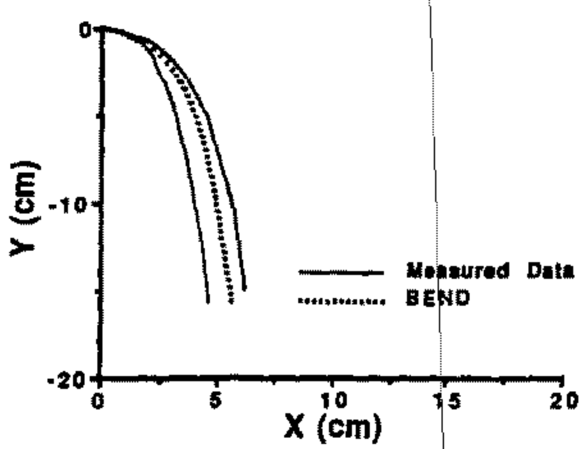

FIGURE 11. Comparison of bending shapes obtained by experimental observation using KES data and our method, fabric C.

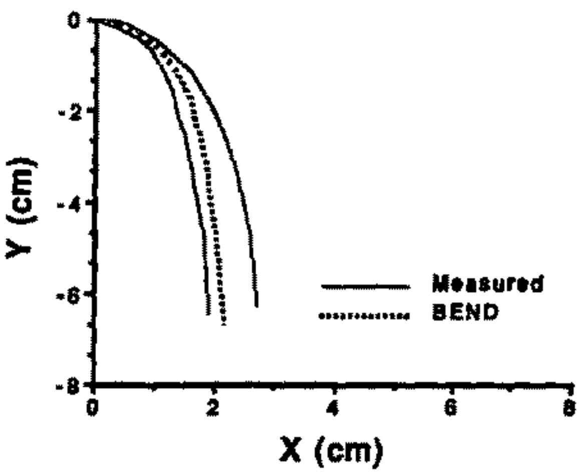

FIGURE 10. Comparison of bending shapes obtained by experimental observation using KES data and our method, fabric B.

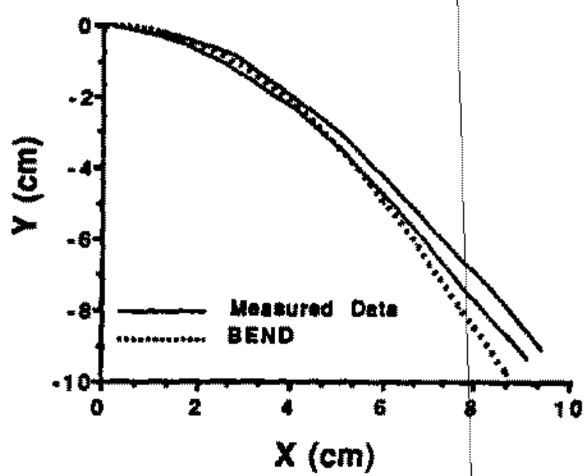

FIGURE 12. Comparison of bending shapes obtained by experimental observation using KES data and our method, cellulose film (D).

the measured data in the upper portion of the fabric, where the value of curvature is relatively large. In the lower portion, where the value of curvature is relatively small, the calculated shape is slightly off the measured shapes. Of these four materials, the film has the largest value of bending rigidity; the result is that in the lower portion of the film near the free end, it is practically straight. It is this fact that makes it very difficult to accurately approximate this moment-curvature relationship for the film in the region of low curvatures.

The good comparisons in Figures 9 through 12 provide supporting evidence that our method is a reliable tool to indirectly measure the nonlinear moment-curvature relationship for a range of fabrics or other materials. Also, it can be used to predict the fabric bending curves.

Conclusions

We have developed a method to indirectly measure moment-curvature relationship using the data of the deformed fabric shape in Cartesian coordinates recorded during a cantilever beam test. In this method, we twice applied numerical techniques of least squares polynomial regression and first-order numerical differentiation. The results of numerical simulation show that in an ideal case, the predicted fabric moment-curvature relationship would be very close to the true solution if the input data of fabric coordinates had enough accuracy. In an actual case, this method may yield some differences since the input data will contain some initial errors. When compared with the experimental observations, however, our method gives a satisfactory performance in predicting fabric bending curves.

In comparison with the standard cantilever beam test to determine the bending rigidity of a fabric, a significant advantage of our method is the clear evidence and prediction of the nonlinear bending behavior of a fabric elastica. Although our method may have limited application for very stiff fabrics, it is important to note that in practical applications, this method presents a valuable new tool to predict the nonlinear moment-curvature relationship and the associated nonlinear bending stiffness.

ACKNOWLEDGMENT

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