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Thermal Stress Analysis of a Bimaterial Strip Subject to an Axial Temperature Gradient

Formulas are derived for the stresses and displacements in a bimaterial strip which is subjected to a temperature gradient that varies linearly in the longitudinal direction. The solution is obtained by superposing certain fundamental linear elastic stress states which are compatible with bar and beam theory. The analytical results are approximate in the sense that stress-free boundary conditions are not satisfied exactly, since only zero resultant force and moment conditions are enforced. Finite element calculations have been performed to verify the results and to ascertain the level of stress concentration near the ends of the strip.

Introduction

It has long been known that structures subjected to nonuniform temperature change, or structures constructed by bonding two or more materials and then subjected to temperature change (be it spatially uniform or nonuniform), will be in a state of thermal stress. Many investigators have presented stress calculations for these types of structures (Born and Horvay (1955), Durelli and Tsao (1955), Goldberg (1953), Goodier (1936)). Recently, much attention has been focused on the problem of predicting the thermal and residual stresses in multilayered electronic components Barnett (1986), Evans and Hutchinson (1984), Glang, et al. (1965), Hu (1979), Isomae (1981), Vilms and Kerps (1982), Suhir (1986, 1988)). Each of these investigations addressed the stresses in the structure during spatially uniform heating or cooling. Timoshenko's (1925) classic paper investigated the similar case of bi-metal thermostats subjected to a spatially uniform temperature change. In his analysis, Timoshenko acknowledged the existence of certain interfacial shear and normal "edge-effect" stresses, but did not provide solutions for them. Suhir (1986, 1988) recently presented solutions for the stresses in the bi-metal thermostat and multi-layer thin-film configurations that enable the investigator to approximate the shear and peeling stresses at the interface of the two metals.

The present paper presents formulas for the deformation and stress state in a bimaterial strip subjected to a temperature gradient which varies linearly in the longitudinal direction. The solution of this problem is obtained with the use of bar and beam theory, together with basic elements from the theory of elasticity. As stated above, previous analyses dealt only with a spatially uniform temperature change. The linear temperature variation assumed in this work is meant as an extension of earlier investigations, and as a first step in understanding the effects of nonuniform heating or cooling. Materials are assumed isotropic, and the physical properties (Young's modulus and the coefficient of thermal expansion) are assumed to have unique values in each layer.

The solution consists of the superposition of three distinct deformation/stress analysis problems. The first step is to separate the layers of the structure, allowing each layer to deform independently due to the assumed temperature gradient. In the second step a set of elastic displacements is imposed on each layer eliminating the relative displacement occurring in the first step. The third step then involves the application of a set of forces and bending moments which result in zero net resultant force and moment on the free surfaces of the structure. It is important to note that the solutions for the stresses and are only approximate, since the third step requires only *resultant* forces and moments at the free surfaces be zero, and not the *stresses* themselves. Thus, the solution for the strip is accurate within bar and beam theory ("strength of materials"), but does not satisfy elasticity theory exactly in the region near the ends of the strip. However, the results are useful in obtaining preliminary design estimates of the stress and deformation states. Furthermore, the results of the analysis have been specialized for the case of a thin film/substrate configuration.

Description of the Problem

Consider the bimaterial strip of total length $2L$ shown in Fig. 1, consisting of a film bonded to a substrate. The ensuing analysis places no restrictions on the relative thicknesses of the two layers, however, ultimately the technologically important configuration of a "thin" film bonded to a much thicker substrate will be of interest. A Cartesian coordinate system is chosen such that the xz plane (location unknown a priori) defines the neutral surface of the strip, and the yz plane is located at a distance equal to L from either end. The thicknesses of the film and substrate are denoted by t_f and t_s, respectively, while the total thickness of the strip is t. A subscript (or superscript) f will be used to denote dimensions, material prop-

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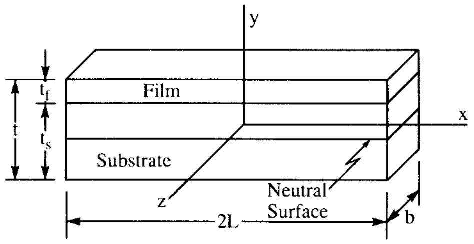

Fig. 1 **Geometry and dimensions of the bimaterial strip**

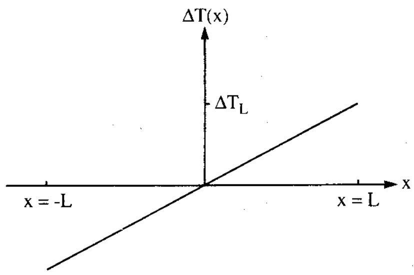

Fig. 2 **Temperature distribution assumed for thermal stress analysis**

erties, stresses, or deflections associated with the film and s will be used for those quantities associated with the substrate. The width of the strip in the z direction is b, consequently the cross-sectional area of the bimaterial strip is bt. Attention will be focused on a long narrow strip where $b, t << L$. Thus, the strip will be expected to deform according to beam theory and be in a state of plane stress respecting the xy plane, thus, $\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0$. In addition, according to the assumptions of bar and beam theory, the normal stress $\sigma_{yy} = 0$. Two material properties enter the analysis. These are Young's modulus E, and the coefficient of thermal expansion (CTE) α.

The most general mathematical expression for a linear temperature gradient in the x direction is

$$\Delta T(x) = \Delta T_0 + \Delta T_L \frac{x}{L} \quad (1)$$

i.e., a linear combination of a constant term and a term proportional to x. The symbols ΔT_0 and ΔT_L represent constants. Timoshenko (1925) has solved the problem dealing with a spatially uniform temperature change, whereby $\Delta T_L = 0$. For this study, the stresses and displacements arising from the second term in equation (1) will be investigated, i.e.

$$\Delta T(x) = \Delta T_L \frac{x}{L} \quad (2)$$

Thus, the results of this work and Timoshenko's analysis could be superposed to yield the solution to a problem where the temperature varied according to equation (1). Figure 2 depicts the temperature distribution assumed for the present analysis.

The primary objective now is to derive formulas for the transverse deflection in each layer as well as the longitudinal and shear stress components.

Solution Formulation

A combination of classical bar and beam theory together with basic elements of elasticity theory is used to solve this problem. The solution will be obtained by superposing certain fundamental linear elastic stress and deformation states whose sum results in proper displacement and stress continuity across the film/substrate interface and force/moment resultant-free conditions on the end cross sections of the strip (i.e., at $x = \pm L$).

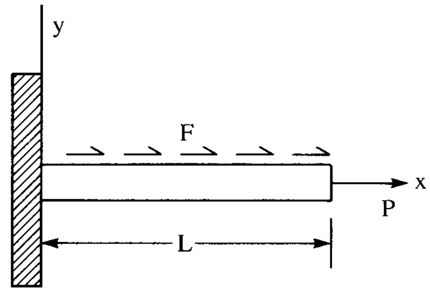

Fig. 3 **Loadings on cantilever beam**

Step 1. The first fundamental solution involves considering the film and substrate as separate layers subjected to the linear temperature distribution given above. Since each layer has a unique CTE, each experiences a unique thermal strain,

$$\epsilon_f^{th} = \alpha_f \Delta T_L \frac{x}{L} \quad \epsilon_s^{th} = \alpha_s \Delta T_L \frac{x}{L} \quad (3, 4)$$

The resulting axial displacements $u^{th}(x)$ are

$$u_f^{th}(x) = \left(\frac{\alpha_f \Delta T_L}{2L}\right) x^2 \quad u_s^{th}(x) = \left(\frac{\alpha_s \Delta T_L}{2L}\right) x^2 \quad (5, 6)$$

where it has been assumed that $u_f^{th}(0) = u_s^{th}(0) = 0$. Notice that equations (5, 6) indicate a relative displacement between the layers as a result of the different CTE's.

Step 2. Ultimately, the deformation of the two layers must be compatible, requiring that the relative displacement between the two layers introduced in Step 1 be eliminated by appropriate superposition. Accordingly, an additional set of elastic displacements, $u_f^e(x)$ and $u_s^e(x)$, must be imposed such that

$$u_f^{th}(x) + u_f^e(x) = u_s^{th}(x) + u_s^e(x) \quad (7)$$

Rearranging this equation and substituting equations (5, 6) for $u_f^{th}(x)$ and $u_s^{th}(x)$ gives

$$\left(\frac{\Delta T_L(\alpha_s - \alpha_f)}{2L}\right) x^2 = u_f^e(x) - u_s^e(x) \quad (8)$$

Thus, an appropriate set of mechanical loadings must act on each layer to produce elastic displacements proportional to x^2 only. It can be shown that, for a cantilever beam with a uniformly distributed shear force F (per unit length) applied to either the upper or lower lateral surface and a compressive axial load $P = FL$ applied on the end, the axial deflection of the centerline is (see Fig. 3)

$$u(x) = -\frac{Fx^2}{2AE} \quad (9)$$

where A is the cross-sectional area of the beam. This loading combination and corresponding displacement suggests how to remove the relative deflection between the film and substrate layers. Assuming that F acts to the right on the upper surface of the substrate and to the left on the lower surface of the film, the elastic displacements in each layer are

$$u_f^e(x) = \frac{Fx^2}{2A_f E_f} \quad u_s^e(x) = -\frac{Fx^2}{2A_s E_s} \quad (10, 11)$$

Substituting these equations back into equation (8) allows F (and hence P) to be determined explicitly,

$$F = \frac{\Delta T_L(\alpha_s - \alpha_f) b E_f t_f E_s t_s}{L(E_f t_f + E_s t_s)} \quad (12)$$

Figure 4 shows the layers as they now appear after subjected to: (i) thermal loading from Step 1, (ii) mechanical forces F and P from Step 2. The axial deformations of the *centerlines* of each layer are now compatible. However, the transverse deformations of the film and substrate layers are still not

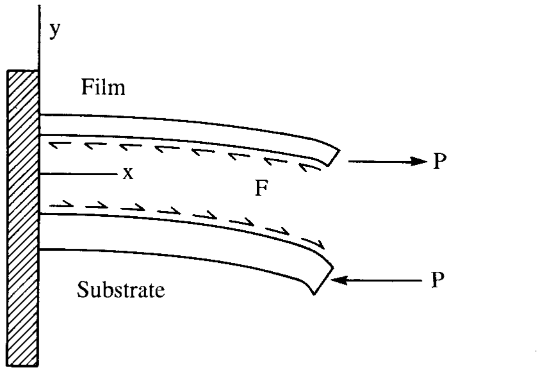

Fig. 4 Deformation state of the film and substrate after application of *F* and *P*

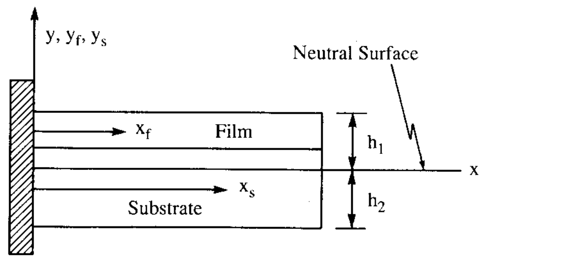

Fig. 5 Local and global coordinate systems, location of neutral surface

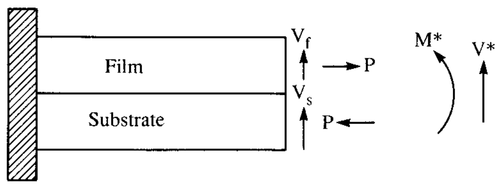

Fig. 6 Resultant forces and moment on composite strip

compatible. The presence of the distributed force F results in bending of the two layers. It can be shown that shear forces of proper magnitude applied at the ends of each layer will negate the transverse deflections due to F. These shear forces are

$$V_f = \frac{Ft_f}{2} \quad V_s = \frac{Ft_s}{2} \tag{13, 14}$$

At this stage the longitudinal and transverse deflections at all points in both layers are zero, and hence the deformations may be considered compatible. However, the resultant forces and moment on the ends of the strip are not zero. Thus, to complete the solution of the problem, this fact must be addressed.

Before proceeding, the stresses in each layer will be summarized. To do so a local coordinate system in each layer is defined as in Fig. 5. Take the film layer as an example. The origin of the y_f coordinate is at $x_f = 0$ and the x_f axis coincides with the neutral axis (the geometric center) of the film layer. The axial force P applied at the end of the film layer results in a tensile axial stress, while the distributed force F results in a compressive axial stress. It should be clear that the bending stress created by F is exactly cancelled by that created by V_f. The total axial stress σ_{xx}^f is calculated by taking the derivative of the axial displacement with respect to x and multiplying the result by E_f. The axial stress in the substrate is calculated similarly. The result of these calculations is

$$\sigma_{xx}^f(x) = \frac{Fx}{bt_f} \quad \sigma_{xx}^s(x) = -\frac{Fx}{bt_s} \tag{15, 16}$$

Shear stresses arise in the film and substrate layers due to the application of F, V_f, and V_s. These shearing stresses may be obtained most easily with aid of the stress equilibrium equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \tag{17, 18}$$

The shear stress in the film is obtained by inserting equations (15, 16) in turn into equation (17), performing the necessary differentiation and integration, and then using the condition that the shear stress at $y_f = (-t_f/2)$ must equal $(+F/b)$. The shear stress in the substrate is calculated similarly. The result of these calculations is

$$\sigma_{xy}^f = \frac{F}{b}\left(\frac{1}{2} - \frac{y_f}{t_f}\right) \quad \sigma_{xy}^s = \frac{F}{b}\left(\frac{1}{2} + \frac{y_s}{t_s}\right) \tag{19, 20}$$

With this construction, equation (18) is satisfied identically. The next step involves eliminating the resultant forces and moment on the end of the composite strip which were introduced during Step 2.

Step 3. As mentioned previously, at this stage the film and substrate layers are undeformed (but not stress-free!) when acted upon by the thermal and mechanical loads considered in the first two steps. Subsequent analysis requires that the bimaterial strip be treated as a composite structure as regards deformations and bending/shearing stress distributions. In particular, the location of the neutral surface of the composite strip is needed as well as expressions for the local x_f, y_f and x_s, y_s coordinates of the film and substrate in terms of the coordinates x, y. The distances h_1 and h_2 shown on Fig. 5 locate the neutral surface of the composite strip using the condition that

$$E_f \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{h_1 - t_f}^{h_1} y\,dy\,dz + E_s \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-h_2}^{h_1 - t_f} y\,dy\,dz = 0$$

$$\text{(zero net axial force)} \tag{21}$$

The result is

$$h_1 = t - h_2 \quad h_2 = \frac{E_f t_f (t_f + 2t_s) + E_s t_s^2}{2(E_f t_f + E_s t_s)} \tag{22, 23}$$

The local coordinates x_f, y_f and x_s, y_s introduced earlier can be expressed in terms of the coordinates x, y via the following equations

$$x_f = x, \quad y_f = y - \left(h_1 - \frac{t_f}{2}\right) \quad \Big| \quad x_s = x, \quad y_s = y + \left(h_2 - \frac{t_s}{2}\right) \tag{24–27}$$

Attention may now be focused back on the resultant forces and moments present on the end of the strip. In order to maintain consistency with bar and beam theory, the *resultant* axial force, shear force, and bending moment on the end of the composite strip must vanish for purely thermal loading. This requirement is consistent with a "strength of materials" analysis, but inconsistent with a formal elasticity solution which would require vanishing *stresses* on the end cross section. An elasticity solution would be exceedingly difficult to accomplish in closed form. Therefore, the approximate solution afforded by resultant theory will be accepted. The elimination of axial force, shear force, and bending moment resultant quantities will now be considered in turn.

Consider the axial forces on the end of the strip. The same force P has been applied to both the film and the substrate, the only difference being that P acts in opposite directions. Thus, the resultant axial force is automatically zero and no additional forces are required. However, since P was applied in opposite directions, there is a resultant moment on the end cross section (see Fig. 6) equal to $Pt/2 = FLt/2$. In order to enforce the zero resultant moment condition, a moment

$$M^* = FLt/2 \tag{28}$$

must be applied in the direction shown in Fig. 6. Next consider the shear forces V_f and V_s on the end of the composite strip. An additional shear force V^* (see Fig. 6) must be applied in order to enforce the zero resultant shear force condition. Thus, the following condition must be enforced

$$V^* = -(V_f + V_s) = -\frac{Ft}{2} \tag{29}$$

The negative sign indicates that V^* acts downward. The presence of the bending moment M^* and shear force V^* give rise to additional stresses in the composite strip. The total bending moment M^T acting on the composite due to M^* and V^* is (positive M^T compresses bottom fibers of the strip)

$$M^T = -\frac{Ftx}{2} \tag{30}$$

The bending moment M^T acting on any cross-section of the strip must equal the bending moment due to the resultant of the stresses σ^f_{xx} and σ^s_{xx}, viz

$$M^T = -\frac{E_f}{\rho}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{h_1 - t_f}^{h_1} y^2 dydz - \frac{E_s}{\rho}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-h_2}^{h_1 - t_f} y^2 dydz \tag{31}$$

where $1/\rho$ is the curvature of the neutral surface. The integrals in the equation above represent the moments of inertia of the film and the substrate about the neutral axis of the composite strip. The bending moment is then,

$$M^T = -\frac{1}{\rho}(E_f I_{fc} + E_s I_{sc}) \tag{32}$$

where I_{fc} and I_{sc} are obtained using the parallel-axis theorem,

$$I_{fc} = \frac{bt_f^3}{12} + bt_f\left(h_1 - \frac{t_f}{2}\right)^2$$

$$I_{sc} = \frac{bt_s^3}{12} + bt_s\left(h_2 - \frac{t_s}{2}\right)^2 \tag{33, 34}$$

Substituting equation (30) into equation (32) yields the expression for curvature

$$\frac{1}{\rho} = \frac{Ftx}{2(E_f I_{fc} + E_s I_{sc})} \tag{35}$$

The corresponding bending stresses are

$$\sigma^f_{xx} = -\frac{FtE_f xy}{2(E_f I_{fc} + E_s I_{sc})} \quad \sigma^s_{xx} = -\frac{FtE_s xy}{2(E_f I_{fc} + E_s I_{sc})} \tag{36, 37}$$

The total axial stress in the film and substrate layers is then obtained by adding equations (36, 37) to equations (15, 16), respectively

$$\sigma^f_{xx} = Fx\left(\frac{1}{t_f b} - \frac{tE_f y}{2(E_f I_{fc} + E_s I_{sc})}\right) \tag{38}$$

$$\sigma^s_{xx} = -Fx\left(\frac{1}{bt_s} + \frac{tE_s y}{2(E_f I_{fc} + E_s I_{sc})}\right) \tag{39}$$

Note here that the axial stress in each layer varies linearly with both x and y and that the stress is equal to zero at the middle ($x = 0$) of the strip.

The total shear stress in the film and substrate is easily obtained using the stress equilibrium equation, equation (17), and imposing the conditions that $\sigma^f_{xy}(x, h_1) = \sigma^s_{xy}(x, -h_2) = 0$. The result of this calculation is

$$\sigma^f_{xy} = -\frac{F}{b}\left(\frac{y - h_1}{t_f} + \frac{bE_f t(h_1^2 - y^2)}{4(E_f I_{fc} + E_s I_{sc})}\right) \tag{40}$$

$$\sigma^s_{xy} = \frac{F}{b}\left(\frac{y + h_2}{t_s} - \frac{bE_s t(h_2^2 - y^2)}{4(E_f I_{fc} + E_s I_{sc})}\right) \tag{41}$$

The expressions for the axial stress and the shear stress in each of the layers are now complete. Only the expression for the lateral deflection of the composite strip remains to be derived. It is derived by integrating the expression for curvature (equation (35)) twice, and imposing the conditions that $v(0) = \frac{dv(0)}{dx} = 0$.

$$v(x) = \frac{Ftx^3}{12(E_f I_{fc} + E_s I_{sc})} \tag{42}$$

It should be clear that the curvature of the nonuniformly heated strip is not constant over its length. This is in marked contrast to the classic Timoshenko (1925) that predicts a constant curvature for the uniformly heated strip.

While the present results are valid for plane stress, i.e., $\sigma_{zz} = 0$, formulas for the plane strain case may be obtained simply by replacing $E \rightarrow E/(1 - \nu^2)$ and $\alpha \rightarrow (1 + \nu)\alpha$ in each layer, where ν indicates Poisson's ratio. Plane strain conditions would be appropriate should the out-of-plane displacement (z direction) be restricted.

Thin Film Approximation

Bi-material structures such as the one described here are utilized frequently in the microelectronics industry in the form of semiconductor wafers and electronic circuit chips. The distinguishing characteristic of these devices is that they consist of a film layer which is much thinner than the substrate. They are often referred to as "thin film" devices. For cases such as this, where the thickness of the film is (for all practical purposes) negligible in comparison to that of the total thickness, the equations derived previously may be simplified. The thin film application implies that, $t_f << t_s$, and thus, $t \approx t_s$. With these approximations in mind, the equations derived previously for the stresses and the lateral deflection may be simplified. These thin film approximations are presented below.

$$\sigma^f_{xx} \approx \frac{Fx}{bt_f} \tag{43}$$

$$\sigma^s_{xx} \approx -\frac{Fx}{bt}\left(1 + \frac{6y}{t}\right) \tag{44}$$

$$\sigma^f_{xy} \approx -\frac{F}{bt_f}\left(y - \frac{t}{2}\right) \tag{45}$$

$$\sigma^s_{xy} \approx \frac{F}{b}\left[\left(\frac{1}{2} + \frac{y}{t}\right) - 3\left(\frac{1}{4} - \left(\frac{y}{t}\right)^2\right]\right] \tag{46}$$

$$v(x) \approx \frac{Fx^3}{E_s bt^2} \tag{47}$$

where

$$F = \frac{\Delta T_L(\alpha_s - \alpha_f)bE_f t_f}{L} \tag{48}$$

Numerical Examples

Thick-Film Problem (Mo/Al). A bimaterial strip consisting of molybdenum on aluminum and subjected to a uniform temperature change has recently been considered by Suhir (1986). The present work seeks to determine the effects of a

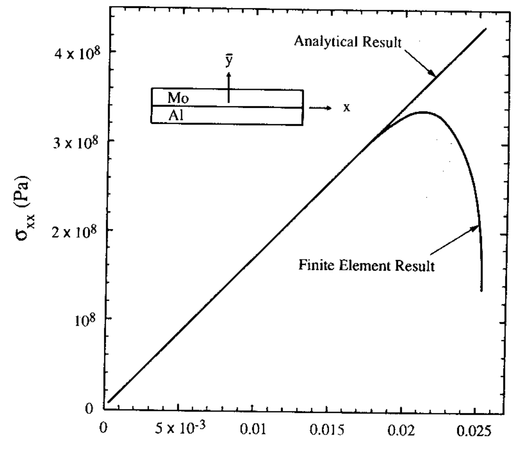

Fig. 7 Axial stress σ_{xx}^{f} in the film at the interface (Mo/Al)

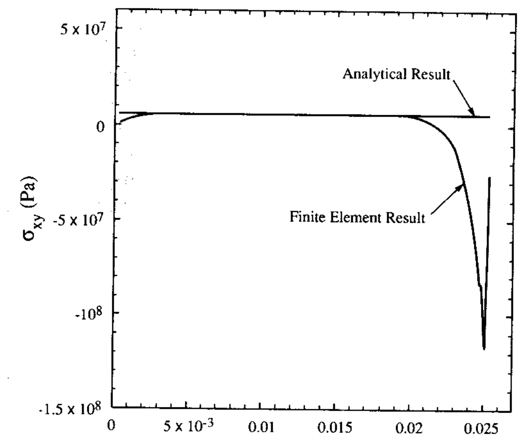

Fig. 9 Shear stress σ_{xy} at the interface (Mo/Al)

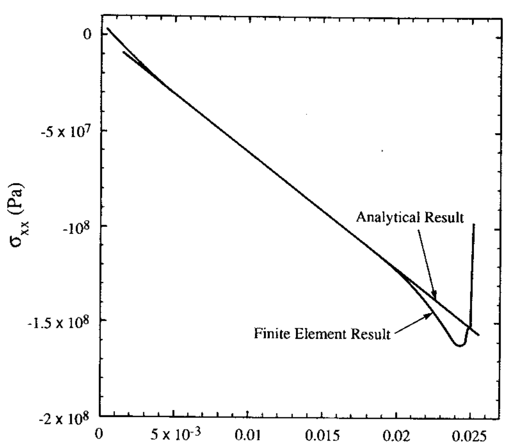

Fig. 8 Axial stress σ_{xx}^{s} in the substrate at the interface (Mo/Al)

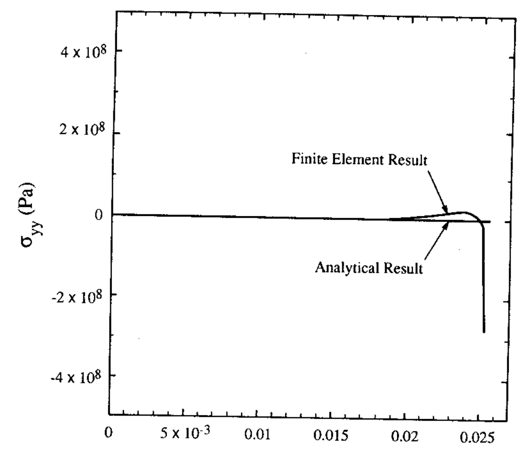

Fig. 10 Normal stress σ_{yy} at the interface (Mo/Al)

linear temperature gradient. The data assumed for this analysis is: $\Delta T_L = 200°C$, $L = 0.0254$ m, $b = 0.001$ m, $t = 0.005$ m.

Substrate-aluminum	Film-molybdenum
$t_s = 0.0025$ m	$t_f = 0.0025$ m
$E_s = 70{,}380 \times 10^6$ Pa	$E_f = 325{,}000 \times 10^6$ Pa
$\alpha_s = 23.6 \times 10^{-6}$ °C^{-1}	$\alpha_f = 4.9 \times 10^{-6}$ °C^{-1}

The results of the analysis are presented as a series of plots showing the stresses, both as a function of the axial coordinate x (Figs. 7–10), and through the thickness as a function of the $\bar{y}$ coordinate (Figs. 11–12). The plots of stress versus x show the analytical solution derived in this paper compared with finite element (FE) analysis results. The FE mesh contained 300 eight node quadrilateral plane stress elasticity elements, with a significant degree of refinement near the free end.

Figures 7 and 8 show the axial stress at the interface in both the film and substrate as a function of distance along the length of the bimaterial strip. The analytical and FE solutions compare very well in the region away from the edge of the strip, i.e., for values of $x < (L - t)$. The analytical solution derived here is not (nor was it intended to be) capable of resolving the true behavior of the stresses in the region very close to the end of the strip. The analytical solution for the axial stress in the film, (Fig. 7) gives a conservative estimate of the maximum value of that stress. Interestingly, the analytical solution for the axial stress in the substrate underestimates the actual maximum compressive stress at the interface (Fig. 8). It is unfortunate that the analytical solution does not "bound" the actual axial stresses at the interface. If this were the case, the simple formulas could be used for design purposes. Numerical experiments show that for configurations involving comparable layer thicknesses, the strength of materials solution for the axial stresses does not err significantly from the actual elasticity solution as computed by FE analysis. The FE results show the tendency for the actual axial stress to drop to zero to satisfy the stress-free edge conditions.

Figures 9 and 10 show the interfacial shear and normal ("peeling") stresses as a function of distance along the length of the bimaterial strip. The shear stress is constant along much of the interface, until just before the end of the strip. Here the shear stress reaches its maximum value and then decays to zero as it must to satisfy the stress-free edge condition. The analytical solution is not capable of predicting this local edge-effect behavior. The peeling stress is essentially zero along much of the interface, then turns slightly tensile and then highly compressive near the end of the strip. In fact, independent analysis (see Bogy (1970)) indicates that this stress component is indeed weakly singular at the free edge. Details of this characteristic are beyond the scope of the present paper but are under active current investigation by the first author.

Figures 11 and 12 show the through-thickness variation of the shear and axial stresses at a distance $x = 0.0127$ m. from the center of the strip. The shear stress is continuous across the interface and varies quadratically through the thickness, while the axial stress is discontinuous and varies linearly.

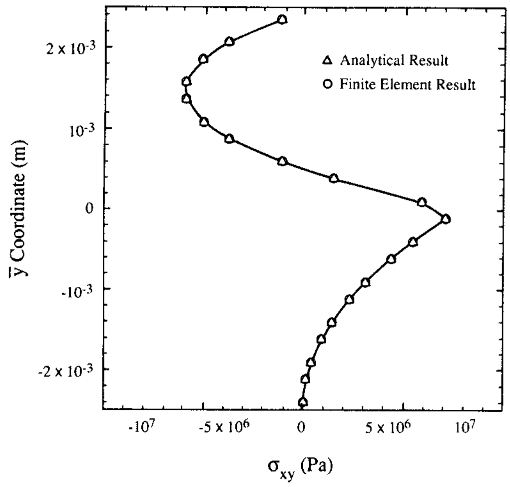

Fig. 11 Distribution of shear stress σ_{xy} through-the-thickness (Mo/Al)

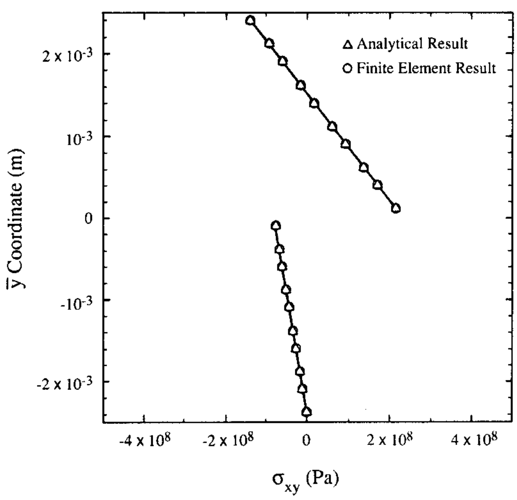

Fig. 12 Distribution of axial stress σ_{xx} through-the-thickness (Mo/Al)

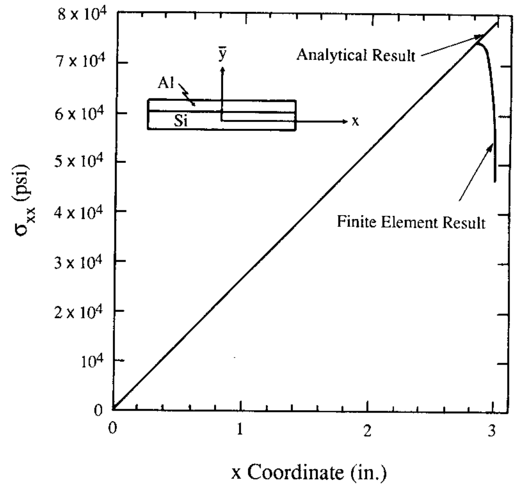

Fig. 13 Axial stress σ^f_{xx} in the film at the interface (Al/Si)

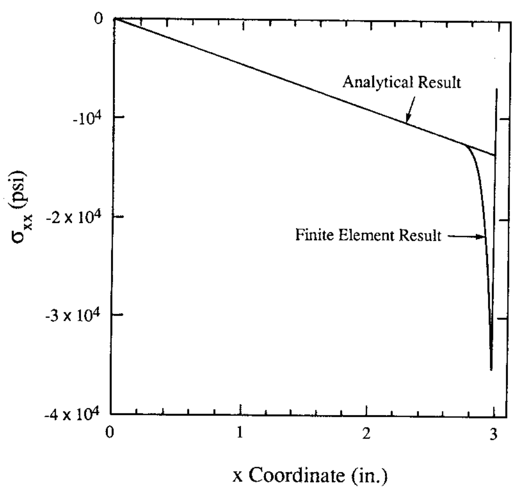

Fig. 14 Axial stress σ^s_{xx} in the substrate at the interface (Al/Si)

The formula for the transverse deflection of the bimaterial strip has also been verified by FE analysis.

Thin-Film Problem (Al/Si). A second numerical example which has been investigated is a bimaterial strip consisting of an aluminum film on a silicon substrate. The distinguishing feature here is the fact that the aluminum layer is relatively thin compared to the silicon substrate. The data assumed for this analysis are: $\Delta T_L = -400°\text{C}$, $L = 3.0$ in., $b = 1.0$ in., $t = 0.25$ in.

Substrate-silicon	Film-aluminum
$t_s = 0.24$ in.	$t_f = 0.01$ in.
$E_s = 24.6 \times 10^6$ psi	$E_f = 10 \times 10^6$ psi
$\alpha_s = 2.5 \times 10^{-6}$ °C^{-1}	$\alpha_f = 23.6 \times 10^{-6}$ °C^{-1}

It is realized that the values assumed for thickness are not representative for realistic electronic structures (e.g., wafers). The values were chosen to avoid the necessity of using an extremely fine mesh, as would be required for vastly different substrate and film thicknesses. The intent of this portion of the paper is to demonstrate the validity of the analytical solutions derived in the paper, an endeavor independent of specific material properties and physical dimensions.

Figures 13 and 14 show the axial stress at the interface in both the film and substrate as a function of distance along the length of the bimaterial strip. Again, the analytical and FE solutions compare very well in the region away from the edge of the strip. The analytical solution for the axial stress in the film (Fig. 13) gives a conservative estimate of the maximum value of that stress. However, the analytical solution for the axial stress in the substrate grossly underestimates the actual maximum compressive stress at the interface (Fig. 14), in the edge-effect zone. Therefore, for thin-film problems, the strength of materials solution is not adequate for predicting axial stress levels.

While not shown here, the interfacial shear stress predicted by the analytical solution agree very closely with FE results in the region away from the ends of the strip.

Conclusions

In conclusion, a solution has been derived which provides an accurate estimate for the lateral deflection of a bimaterial strip subjected to a linear, axial temperature gradient. Also, formulas for the axial and shear stresses in the strip have been obtained which give accurate results in the region away from the end of the strip.

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