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GEOMETRIC NONLINEARITY IN A BIMATERIAL STRIP

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ABSTRACT

A classic paper by Timoshenko in 1925 dealt with thermal stresses in bi-metal thermostats and has been widely used for designing laminated structures, and in contemporary studies of stresses in thin-film electronic devices. The state of stress in such layered structures results from a combination of axial force and bending moment in each of the material layers. This paper explores the possible nonlinear interaction, or membrane stiffness effects in bimaterial strips, and their impact on the interfacial stress state. For the particular material combination investigated here, the stress state is altered measurably. The nonlinear analysis predicts that the interfacial stresses are not necessarily confined to an edge-effect zone, but are active along the entire interface. These findings may be of practical significance for those material interfaces that are very weak in tension, hence susceptible to peeling or delamination.

INTRODUCTION-LINEAR ANALYSIS

The objective of this analysis is to derive the internal force and moment distribution, state of stress, and deformations in the bimaterial strip shown in Fig. 1a. The formulation follows Timoshenko[1] and details are included here to render the paper self-contained.

Attention is focused on a bimaterial strip having length $2L$ and rectangular cross section with dimensions b by h. The thicknesses of the individual layers are h_1 and h_2. The material properties entering the analysis are Young's moduli E_1 and E_2 Poisson's ratios ν_1 and ν_2 and the linear coefficients of thermal expansion α_1 and α_2. Each layer is assumed to experience an equal temperature change ΔT from an undeformed reference state. A local coordinate system has been fixed in each layer, as shown in Fig. 1a.

As discussed by Timoshenko[1], the load transfer between layers occurs at the ends $x=\pm L$ by means of a concentrated interlaminar shear force q and moment m, see Fig. 1b. This loading system leaves the end surfaces of the strip resultant free, as required by a solution consistent with bar and beam theory. The quantities q and m are obtained by enforcing compatibility of longitudinal and transverse displacements $u(x)$ and $v(x)$, respectively, along the interface. These interfacial displacements are:

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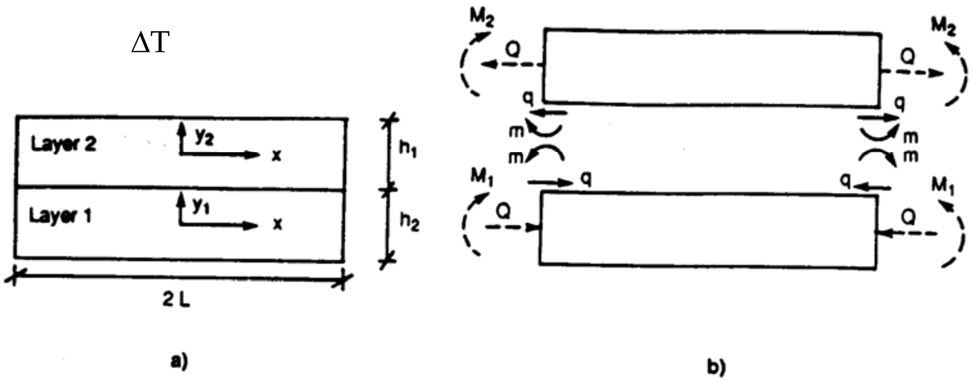

Figure 1-Geometry, Dimensions, and Load Transfer Mechanism for the Bimaterial Strip

<u>Layer 1:</u>

$$u_1(x) = \alpha_1 \Delta T x - \frac{qx}{A_1 E_1} - \left(\frac{qh_1}{2} - m\right)\frac{x}{E_1 I_1}\frac{h_1}{2} \quad (1)$$

$$v_1(x) = \left(\frac{qh_1}{2} - m\right)\frac{x^2}{2E_1 I_1} \quad (2)$$

<u>Layer 2:</u>

$$u_2(x) = \alpha_2 \Delta T x + \frac{qx}{A_2 E_2} + \left(\frac{qh_2}{2} + m\right)\frac{x}{E_2 I_2}\frac{h_2}{2} \quad (3)$$

$$v_2(x) = \left(\frac{qh_2}{2} + m\right)\frac{x^2}{2E_2 I_2} \quad (4)$$

where the section properties A_1, A_2, I_1, I_2 are

$$A_1 = bh_1 \quad I_1 = bh_1^3/12 \qquad A_2 = bh_2 \quad I_2 = bh_2^3/12 \quad (5)$$

The expressions for displacements are consistent with bar and beam theory and do <u>not</u> include any effects of interaction between axial force and bending moment. This interaction will be explored in the next section of the paper. The displacement expressions are valid if the stress normal to the x, y_1 or x, y_2 plane is zero, consistent with generalized plane stress. For plane strain deformation, replace E with $E/(1-\nu^2)$ and α with $(1+\nu)\alpha$ in each layer. Compatibility of displacements along the interface requires that

$$u_1(x) = u_2(x) \quad v_1(x) = v_2(x) \tag{6}$$

Solving for q and m yields

$$q = \frac{(a_1 - \alpha_2)\Delta T}{\left[\dfrac{1}{E_1 A_1} + \dfrac{1}{E_2 A_2} + \dfrac{(h_1 + h_2)^2}{4(E_1 I_1 + E_1 I_1)}\right]} \tag{7}$$

$$m = \frac{(a_1 - \alpha_2)\Delta T}{2\left[\dfrac{1}{E_1 A_1} + \dfrac{1}{E_2 A_2} + \dfrac{(h_1 + h_2)^2}{4(E_1 I_1 + E_1 I_1)}\right]} \frac{h_1 E_2 I_2 - h_2 E_1 I_1}{E_1 I_1 + E_2 I_2} \tag{8}$$

The shear force q acting on the interface and moment m are statically equivalent to concentrated axial force Q acting at the centroidal axis of each layer, together with a moment M_1 (or M_2). This equivalent force system is shown in Fig. 1b. Thus,

$$Q = q \quad M_1 = \frac{q h_1}{2} - m \quad M_2 = \frac{q h_2}{2} + m \tag{9}$$

The stresses in the bimaterial strip are also of practical importance. The only non-zero component of stress predicted by the present theory is σ_{xx}, which is obtained by summing the axial stress produced by Q and bending stress produced by M_1 or M_2.

$$\sigma_{xx1} = -\frac{Q}{A_1} - \frac{M_1 y_1}{I_1} \tag{10}$$

$$\sigma_{xx2} = \frac{Q}{A_1} - \frac{M_2 y_2}{I_2} \tag{11}$$

Another quantity of interest is the transverse displacement at the ends of strip.

$$v_{\max} = v_1(L) = v_2(L) = \frac{(a_1 - \alpha_2)\Delta T L^2}{4\left[\dfrac{1}{E_1 A_1} + \dfrac{1}{E_2 A_2} + \dfrac{(h_1 + h_2)^2}{4(E_1 I_1 + E_1 I_1)}\right]} \frac{h_1 + h_2}{E_1 I_1 + E_2 I_2} \tag{12}$$

The transverse displacement is sensitive only to the value of M_1 or M_2, and is independent of the magnitude of Q. According to the linear theory, the strip deforms in pure bending, i.e. it assumes a constant radius of curvature throughout its length.

This completes the linear analysis of the bimaterial strip problem.

NONLINEAR ANALYSIS

The linear analysis has shown that each layer of the strip is loaded by a combination of axial force and bending moment. In that analysis, the possible interaction between these two loading components was neglected. Purpose of the following analysis is to investigate the significance of these interaction or "membrane stiffness" effects. The following formulas give the transverse deflections of the centroidal axis at the end $x=L$ of each layer including interaction effects[2], assuming again that all load transfer occurs in a region close to the ends of the strip, by way of an interfacial shear force and bending moment

$$v_1(x) = \frac{M_1}{Q}\left(\frac{1}{\cos\left(\frac{Q}{E_1 I_1}\right)^{1/2} L} - 1\right) \tag{13}$$

$$v_2(x) = -\frac{M_2}{Q}\left(\frac{1}{\cosh\left(\frac{Q}{E_2 I_2}\right)^{1/2} L} - 1\right) \tag{14}$$

If the interaction effects are truly negligible, as was assumed in the linear analysis, Eqns. (13)-(14) should predict virtually the same transverse displacements as Eqn. (12) above. After all, as Q goes to 0, these formulas each reduce to Eqn. (12). A numerical example involving a molybdenum/aluminum strip, recently studied by Suhir[3], was employed for the comparison.

<u>Physical Properties</u>

$$\Delta T = 240^o C \quad L = 25.4\text{mm} \quad b = 1.0\text{mm} \quad h = 5\text{mm}$$

<u>Layer 1-Aluminum</u>

$$h_1 = 2.5\text{mm} \quad E_1 = 70{,}380\text{MPa} \quad \nu_1 = 0.3 \quad \alpha_1 = 23.6x10^{-6}\ {}^{\text{o}}\text{C}^{-1}$$

<u>Layer 2-Molybdenum</u>

$$h_2 = 2.5\text{mm} \quad E_2 = 325{,}000\text{MPa} \quad \nu_2 = 0.3 \quad \alpha_2 = 4.9x10^{-6}\ {}^{\text{o}}\text{C}^{-1}$$

The values of the interface shear force and moment predicted by the linear theory are q=336.5N and m=0.271N-m. The transverse displacements for each layer computed via the linear and nonlinear theories are then,

Linear: $v_1(L) = 4.797\text{x}10^{-4}\,\text{m} \quad v_2(L) = 4.797\text{x}10^{-4}\,\text{m}$

Nonlinear: $v_1(L) = 3.902\text{x}10^{-3}\,\text{m} \quad v_2(L) = 4.013\text{x}10^{-4}\,\text{m}$

Clearly, there is a discrepancy between the two theories. Physically speaking, the compressive axial force in layer 1 effectively reduces the bending rigidity, leading to a greater transverse deflection than anticipated by the linear theory. The tensile axial force in layer 2 has the opposite effect. The logical conclusion of this comparison is that the nature of the load transfer is not adequately predicted by the linear theory and that nonlinear geometric (or membrane stiffness) effects must be included in the analysis.

In order to gain further insight into the nonlinear effects, a very simple, yet refined model of the load transfer mechanism was conceived and is shown in Fig. 2.

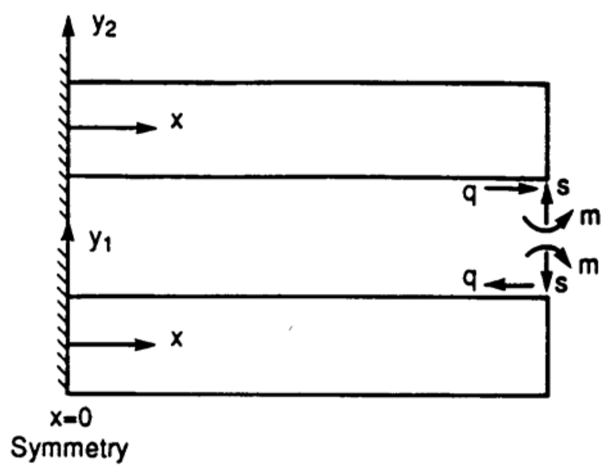

Figure 2-Model of Load Transfer Mechanism

The expressions for the longitudinal and transverse displacements, and cross section rotation of the centroidal axis at the end (x=L) for the two layers are:

$$u_1(L) = -\frac{QL}{A_1E_1} + \alpha_1 \Delta T \qquad (15)$$

$$v_1(L) = \frac{M_1}{Q}\left(\frac{1}{\cos\left(\frac{Q}{E_1I_1}\right)^{1/2} L} - 1\right) - \frac{s}{\left(\frac{Q}{E_1I_1}\right)^{1/2} Q}\left(\tan\left(\frac{Q}{E_1I_1}\right)^{1/2} L - \left(\frac{Q}{E_1I_1}\right)^{1/2} L\right) \qquad (16)$$

$$\theta_1(L) = \frac{M_1\left(Q/E_1I_1\right)^{1/2}}{Q}\tan\left(\frac{Q}{E_1I_1}\right)^{1/2}L - \frac{s}{Q}\frac{1-\cos\left(Q/E_1I_1\right)^{1/2}L}{\cos\left(Q/E_1I_1\right)^{1/2}L} \tag{17}$$

$$u_2(L) = \frac{QL}{A_2E_2} + \alpha_2\Delta T \tag{18}$$

$$v_2(L) = -\frac{M_2}{Q}\left(\frac{1}{\cosh\left(\frac{Q}{E_2I_2}\right)^{1/2}L} - 1\right) + \frac{s}{\left(\frac{Q}{E_2I_2}\right)^{1/2}Q}\left(-\tanh\left(\frac{Q}{E_2I_2}\right)^{1/2}L + \left(\frac{Q}{E_2I_2}\right)^{1/2}L\right) \tag{19}$$

$$\theta_2(L) = \frac{M_2\left(Q/E_2I_2\right)^{1/2}}{Q}\tanh\left(\frac{Q}{E_2I_2}\right)^{1/2}L - \frac{s}{Q}\frac{1-\cosh\left(Q/E_2I_2\right)^{1/2}L}{\cosh\left(Q/E_2I_2\right)^{1/2}L} \tag{20}$$

Compatibility of displacement and rotation at $x=L$ requires that

$$u_1(L) + \theta_1(L)\frac{h_1}{2} = u_2(L) - \theta_2(L)\frac{h_2}{2} \tag{21}$$

$$v_1(L) = v_2(L) \tag{22}$$

$$\theta_1(L) = \theta_2(L) \tag{23}$$

The model is approximate in the sense that all load transfer is assumed to occur at the ends of the strip. Now however, the presence of a transverse shear force *s* acting at the end of each layer is admitted. The resultant force and moment on the end of the composite strip is required to vanish. When the interfacial displacements at the end of strip are then forced to be compatible based on a nonlinear elastic analysis, the interface forces and moment are: *q*=336.2N, *m*=0.298N-m, and s=6.35N. The presence of the shear force *s* is necessary to enforce compatibility of displacements at the interface, when nonlinear effects are included. The major weakness of this simple model is that load transfer is permitted to occur only at the ends of the strip, and not over a finite portion of the interface. Thus there is no indication as to the distribution of the interfacial stress. A second weakness is that compatibility of interface displacements is enforced only at the ends of the strip, not along the entire interface as was possible in the linear analysis.

CONCLUSIONS

The issue of interaction between axial forces and bending moments in biomaterial strips has been investigated. It appears that for the material combination studied in this paper (molybdenum/aluminum), the interaction effects are important. The effect of the interaction is to alter the distribution of stresses on the material interface.

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