IMPULSE DAMPING CONTROL OF AN EXPERIMENTAL STRUCTURE

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1. INTRODUCTION

Early on, in an extension of N. N. Krasovskii's geometric approach to optimal control theory [1], Lucien Neustadt showed that fuel optimal control of dynamic systems is achieved by use of impulsive forces [2]. However, Neustadt was primarily concerned with the formulation of optimal control problems that lend themselves readily to numerical solutions. Since fuel optimal control problems are not easily solved numerically, Neustadt abandoned fuel optimal control in favor of investigating other optimal control problems.

Later on, the impracticality of impulsive controls led Hajek to re-examine fuel optimal control and to introduce bounded control inputs. Unlike the bang–bang nature of minimum time control [3], he showed that bounded fuel optimal control is bang–off–bang [4]. Although bounded fuel optimal control was successfully demonstrated on a variety of second order systems [5–9], bounded fuel optimal control of higher order systems remained plagued with numerical difficulties. Consequently, fuel optimal control was largely abandoned in favor of techniques that offer closed form solutions, such as linear optimal control. A notable exception to this trend is the quantized control scheme developed by Shenhari and Meirovich [10]. Unable to solve the general fuel optimal problem for systems of order 2n, they employed the Independent Modal Space Control method to yield n independent second order minimization problems. These modal fuel optimal control problems were each solved with relative ease. The resulting modal controls were then transformed to yield quantized actuator forces.

More recently, a closed loop approximation to fuel optimal control of oscillatory systems was developed. Known as Impulse Damping Control, this approximation uses properties associated with the fuel optimal control of a harmonic oscillator [11] to formulate a near fuel optimal closed loop control strategy for spacecraft damping [12]. In this scheme, recursive calculations of standard deviations of spacecraft displacements and velocities regulate on–off actuators. This technique was demonstrated numerically using a ten-mode cantilevered beam [13]. Then a single-mode vibration suppression experiment was successfully completed [14]. This paper focuses on the extension of these results to a multi-mode experimental system.

The test article consists of a 16 ft cantilevered beam. The beam is suspended vertically and fitted with on–off air fed valves for actuation.

2. IMPULSE DAMPING CONTROL

The equation of motion for an undamped elastic structure controlled by p discrete propulsive control forces is represented by

$$m(x) \frac{\partial^2 u(x,t)}{\partial t^2} + Lu(x,t) = \sum_{i=1}^{p} F_i(t) \delta(x-x_{ui}),$$

(1)

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in which \( T(x) \) denotes the beam tension. The beam tension is a piecewise continuous function given by

\[
T(x) = \{ m_i(L - x) + M_i + (p - i + 1)M_e \} g, \quad x_{at(i-1)} \leq x \leq x_{at(i)}, \quad i = 1, 2, \ldots, p + 1. \tag{6}
\]

Strain gages are employed to obtain distributed approximations of the structure's displacement and velocity. A total of \( r = 8 \) strain gages are distributed along the beam centerline, beginning at the root. The strain gage signals are processed in an Intel 80386 microprocessor, where the displacements are approximated using cubic spline functions of the form

\[
u(x, t) = a_i(t)x^3 + b_i(t)x^2 + c_i(t)x + d_i(t), \quad x_{gt} \leq x \leq x_{gt(i+1)}. \tag{7}\]

Each spline is associated with a subdomain, with its boundaries defined by the gage locations. The spline coefficients are determined from the strain gage measurements, which are related to the beam curvature at the subdomain boundaries by the geometric relationship

\[
u''(x_{gt}, t) = -2 \varepsilon(x_{gt}, t)/h, \tag{8}\]

in which \( \varepsilon(x_{gt}, t) \) is the strain and \( h \) is the beam thickness. Also, necessary in determining these coefficients are the beam's boundary conditions \( u(0, t) = u'(0, t) = u'(L, t) = u''(L, t) = 0 \) and the subdomain compatibility conditions

\[
u(x_{gt}, t) = u(x_{gt}^+), \quad u'(x_{gt}, t) = u'(x_{gt}^+), \quad u''(x_{gt}, t) = u''(x_{gt}^+), \quad i = 1, 2, \ldots, r. \tag{9}\]

The beam's velocity is approximated by the backward finite difference

\[
\dot{u}(x, t) \approx (u(x, t) - u(x, t - \Delta t))/\Delta t. \tag{10}\]

Note that small inaccuracies in strain measurements lead to errors in the beam displacement which in turn result in larger errors in the beam velocity. However, this does not directly affect the control law since equation (3) depends only on the sign of the velocity and not its magnitude.

Each actuator consists of two (± directions) air-fed solenoid valves capable of producing 0.35 lb of thrust. A total of four actuators are used in the experiment, with a single line supplying the air for the entire system. The actuator valves are opened and closed in accordance with equation (3). One such actuator is shown in Figure 1.

Figure 1. Beam section with third actuator.
modes become more prevalent, the control requires rapid switching as the sign of the velocity quickly vacillates. The control continues for approximately 13.5 seconds, at which time the motion is contained in the prescribed deadband of 0.5 inches.

The results of a similar test, but with $\mu_u=0.5$, are shown in Figure 4. Again shown are the displacement at the fourth actuator location, the standard deviation of the displacement scaled by $\mu_u$, and the actuator state. The reduction of $\mu_u$ decreased the time during which the actuators are fired and dramatically reduced the decay rate. For this case, $\gamma \approx 0.72$, almost 40% less than the previous case. Consequently, the control required almost 24 seconds to drive the beam’s motion to the deadband.

Next, we again let $\mu_u=1$, but the supply pressure was reduced to 20 psi. The results shown in Figure 5 reveal an even lower decay rate of $\gamma \approx 0.63$, or 45% less than with $P_s=35$ psi and $\mu_u=1$. This case required almost 30 seconds to drive the motion to the deadband. Clearly, variations in either $P_s$ or $\mu_u$ can have a dramatic effect on the system response.
single supply line, the failure of some of the actuators meant that more supply air was available for the remaining actuators. Thus, the properly functioning actuators were able to exert larger control forces. Consequently, the response of the structure is similar in both cases.

5. CONCLUDING REMARKS

Since the fuel optimal control problem cannot be exactly solved for even moderately complicated systems, other approaches must be used. This paper extends the characteristics associated with the fuel optimal control of a harmonic oscillator to develop a near-minimum fuel control algorithm for the vibration suppression of spacecraft. Recursive determinations of the standard deviations of displacement and velocity govern the operation of single level thrusters, resulting in a bang–off–bang controller.

The control was demonstrated on a vertically suspended 16 ft cantilevered beam. The structure’s response was easily manipulated by minor alterations in the control law. Furthermore, the control system performance was not seriously degraded in the presence of multiple actuator failures.

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REFERENCES