LETTERS TO THE EDITOR


Of primary interest is the performance of the control in the presence of multiple mode participation. This was demonstrated with $\mu_a = 0.5$ and $P_s = 35$ psi. The results of this case are displayed in Figure 6. The standard deviation was again computed with a window size equal to one-half the period of the first mode. The actuator state reveals that this case required more complicated switching than the previous cases. When the displacement was within its scaled standard deviation, rapid changes in the velocity direction required equally rapid switches in the actuator state. However, this rapid switching did not adversely affect the system. The control was able to restrict the motion to a 0.35 inch deadband within 17 seconds.

The previous case was reconsidered in the presence of actuator failure. Throughout the test, actuator 2 failed completely while actuators 1 and 4 operated in only one direction. Actuator 3 remained fully operational. The displacement at the fourth actuator location is shown in Figure 7. Since the control fulfilled the objective in approximately 18 seconds, it appears that failing the actuators did not seriously degrade the system performance. This is a consequence of the experimental setup. Because all the actuators operated off a
The structure was discretized using eight global admissible functions resulting in four converged modes. Table 2 gives the four lowest frequencies of the structure and the modeled mode shapes are shown in Figure 2. Agreement between the model frequencies and the experimental frequencies attest to the validity of the mathematical model.

![Mode frequencies](image)

Figure 2. Lowest four computed modes of vertically suspended beam.

4. EXPERIMENTAL RESULTS

The governing control events of equation (2) are easily adapted to satisfy specific hardware elements such as the strain gages used in this experiment. As described in the previous section, small errors in the strain measurements result in large errors in the determination of the beam’s velocity. Consequently, we set $\mu_s = 0$ for all of the cases considered to make $E_s$ a passive constraint. In contrast, the use of accelerometers in the sensing system would likely yield velocities that are more accurate than displacements. In this case, it may be desirable to set $\mu_s = \infty$ to make $E_1$ passive.

We first consider the case of single mode participation to determine how $\mu_u$ and the actuator supply pressure ($P_u$) affect the system response. Figure 3 shows the displacement of the structure at the fourth actuator ($x = 14.8$ ft) in response to first mode excitation. Both the controlled and uncontrolled responses are given for comparison. In the controlled case, all actuators were supplied with a single supply line with $P_u = 35$ psi. The state of the fourth actuator is also shown in Figure 3. Note that the actuator is only fired when the displacement is within the bounds of the scaled standard deviation, computed with a window size $T_u = 1.9$ seconds (one-half the period of the first mode) [17]. For this case, $\mu_u = 1.0$, which resulted in a linear decay rate of $\gamma \approx 1.16$. The actuator state shows that in the presence of only the first mode, a smooth switching sequence results. As the higher
in which \( m(x) \) denotes the mass distribution, \( \mathcal{L} \) is a positive definite differential operator representing the system stiffness, and \( F(t) \) represents the control force applied at \( x_{at} \). It has been established for the oscillator that the fuel optimal control force is obtained by activating the actuator when the system undergoes a maximum velocity and minimum displacement [11–13]. Applying the same philosophy to each of the structure’s actuators, we define the Impulse Damping Control events

\[
E_1(x_{at}) : |\dot{u}(x_{at}, t)| \leq \mu_o \sigma_o(x_{at}, t), \quad E_2(x_{at}) : |\dot{u}(x_{at}, t)| > \mu_o \sigma_o(x_{at}, t),
\]

\( \mu_o \) and \( \mu_o \) are designer selected scaling factors. Also, the running-in-time standard deviations of displacement and velocity at the actuator location are denoted by \( \sigma_o(x_{at}, t) \) and \( \sigma_v(x_{at}, t) \), respectively. The decentralized control law is given as [12, 13]

\[
F(t) = \begin{cases} 
-F_{max} \text{ sgn } (\dot{u}(x_{at}, t)), & \quad E_1(x_{at}) \cap E_2(x_{at}), \\
0, & \quad \text{otherwise.}
\end{cases}
\]

The actuator forces governed by equation (3) are applied opposite the local velocity when the structure’s local displacement is near zero and the velocity is near a peak. This bang–off–bang control law eliminates the need for variable thrust actuation. Note that the control of equation (3) yields quantized controls in the modal space. Furthermore, the decentralized nature of this control law guarantees attractive robustness characteristics [15, 16].

3. EXPERIMENTAL SET-UP

A vertically suspended cantilevered beam fitted with a tip mass was designed and fabricated (see Table 1). The beam was suspended vertically to eliminate problems associated with beam buckling. Neglecting necessary wires and hoses, the mass distribution is expressed as

\[
m(x) = m_b + M_t \delta(x - L) + M_a \sum_{i=1}^{p} \delta(x - x_{at}),
\]

where \( m_b \) is the beam mass per unit length, \( M_t \) is the tip mass and \( M_a \) is the mass of each of \( p = 4 \) actuators. This configuration results in a stiffness operator of the form

\[
\mathcal{L} = \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2}{\partial x^2} \right) - \frac{\partial}{\partial x} \left( T(x) \frac{\partial}{\partial x} \right),
\]