


V. Strain Measurement

The open-loop fuel-optimal control solution does not require measurements of the states. However, measurements of the states were performed for purposes of comparison with the analytical predictions. Indeed, all of the states can be extracted from the strain measurement and the angular rate measurement.

A strain gauge was placed where the beam joins the mount. The gauge was thin, flexible, and self-temperature compensating. A high resistance of 350 Ω was selected to increase the signal-to-noise ratio. The transverse sensitivity was low (0.8 ± 0.2%) to reduce the corruption of the pure bending component by torsional effects. The elastic displacement of the beam relative to a line tangent to the beam at its root is expressed in the form of a cubic function \( w_m(x, t) \), and we impose the boundary conditions

\[
\frac{\partial w_m(0, t)}{\partial x} = \frac{\partial^2 w_m(L, t)}{\partial x^2} = 0
\]

where \( \Gamma(t) \) is the beam curvature at the root. We obtain the form

\[
w_m = -\frac{\Gamma}{6L} x^3 - \frac{e}{\Gamma} x^2, \quad \Gamma = -\frac{2e}{T}
\]  

where \( e = \epsilon(t) \) is the measured strain. The time rate of change of \( w_m(x, t) \) was computed by a backward finite difference.

VI. Angular Rate Measurement

The angular rate transducer located on the mount provided voltage output linearly proportional to the angular rate input. Hysteresis was essentially zero with a slow transient shift of the null signal typically ±1% of full scale. The 12-bit data acquisition restricted input resolution to 1.221 × 10^{-3} deg/s. The unit threshold was approximately 0.01 deg/s with a maximum measured rate of 180 deg/s. Angular position measurements were extracted by numerical integration.

VII. Experimental Results

The analytically predicted natural frequencies and the measured natural frequencies are compared in Table 3. The experimentally measured frequencies were obtained by imparting initial conditions on the structure that excited the rigid-body mode and several flexible-body modes. Free response angular rate measurements were used to calculate the power spectral density shown in Fig. 8.

The modal coordinates are extracted from the measured quantities in a procedure that was generalized for maneuvering elastic bodies in Ref. 12. More specifically, we can transform the measured quantities into modal quantities from Fig. 9. We obtain

\[
\theta(t) = \theta_m(t) - \frac{1}{x_c} \int_0^L \frac{1}{M} \rho(x) w_m(x, t) \, dx
\]  

\[
w(x, t) = w_m(x, t) - \frac{x}{x_c} \int_0^L \frac{1}{M} \rho(x) w_m(x, t) \, dx
\]

where \( x_c = 19.94 \) in. denotes the position of the mass center, \( M = 0.358 \) slugs denotes the total mass, \( \theta_m(t) \) is the measured angle, and Eq. (23) is substituted into the integral terms before they are evaluated.

The hinged-free beam is skewed 35 deg in 6 s. The numerical solution of the fuel-optimal control problem yields

\[
\alpha^* = 12.389098861099480, \quad \text{fuel} = 0.08071612078
\]

and the optimal normal vector

\[
\tilde{\eta} = \begin{bmatrix}
-1.637022271802352 \\
-4.894758065691531 \\
-3.000156016538542 \\
0.195758065691530
\end{bmatrix}
\]

The phase plane plot of the rigid-body rotation angle \( \theta(t) \) is shown in Fig. 10. The phase plane plot of the modal coordi-