Fuel-Optimal Slewing of an Experimental Hinged-Free Beam
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tural damping and air damping, is of the proportional type, thereby preserving the normal mode characteristics of the vibration. This assumption is quite reasonable in the case of light damping.

The air thrust actuators apply point forces \( u_1(t) \) and \( u_2(t) \), and the corresponding spatial impulse functions are \( \delta(x - x_1) \) and \( \delta(x - x_2) \), respectively. The normal modes and frequencies of the system are computed by solving the associated eigenvalue problem

\[
\omega^2 \rho(x) \phi_r(x) = \frac{d^2}{dx^2} \left[ EI \frac{d^2 \phi_r(x)}{dx^2} \right], \quad (r = 0, 1, 2, \ldots)
\]

subject to the boundary conditions just indicated. After one arranges the modes \( \phi_r(x) \) and corresponding frequencies \( \omega_r \) in ascending order, the lowest mode is the rigid-body slewing mode denoted by \( \phi_0(x) \), and the associated frequency is zero \( (\omega_0 = 0) \). The second mode is the fundamental elastic mode denoted by \( \phi_1(x) \) and the associated fundamental frequency is denoted by \( \omega_1 \). The solution of the eigenvalue problem is obtained by the Rayleigh-Ritz method using the admissible functions

\[
\psi_r(x) = \frac{X}{2L} \left[ 1 + \left( \frac{X}{L} \right)^{r-1} \right], \quad (r = 1, 2, \ldots, 12)
\]

The associated mass and stiffness matrices are\(^{10}\)

\[
m_{r}\phi_m \int_0^\infty \psi_r(x) \psi_r(x) \, dx + \rho_b \int_0^\infty \psi_r(x) \tilde{\psi}_r(x) \, dx
\]

\[
k_{r} = E_m I_m \int_0^\infty \psi_r(x) \tilde{\psi}_r(x) \, dx + E_b I_b \int_0^\infty \tilde{\psi}_r(x) \psi_r(x) \, dx
\]

The lowest three modes are shown in Fig. 3. Note that the two air thrust actuators were placed at the two nodes of the third mode to eliminate control spillover into that mode. The placement of the two actuators was an iterative process as a consequence of the effect of the actuator mass on the nodal locations.