Table 1  Control parameters for $\tau = 1.5$

<table>
<thead>
<tr>
<th>$\tau_f$</th>
<th>Optimal fuel</th>
<th>$\tau_f$</th>
<th>$\text{sgn}(\mathcal{g}(\tau_f^*, \tau_f))$</th>
<th>$c_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>6.5940863</td>
<td>0.0000000</td>
<td>-</td>
<td>0.1135749</td>
</tr>
<tr>
<td></td>
<td>1.0622951</td>
<td>+</td>
<td></td>
<td>0.2882963</td>
</tr>
<tr>
<td></td>
<td>2.9377049</td>
<td>-</td>
<td></td>
<td>0.3652354</td>
</tr>
<tr>
<td></td>
<td>4.0000000</td>
<td>+</td>
<td></td>
<td>0.2358035</td>
</tr>
<tr>
<td>6.0</td>
<td>1.5501346</td>
<td>0.2297864</td>
<td>+</td>
<td>0.4366778</td>
</tr>
<tr>
<td></td>
<td>3.3180872</td>
<td>-</td>
<td></td>
<td>0.1137695</td>
</tr>
<tr>
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<td>6.0000000</td>
<td>+</td>
<td></td>
<td>0.4483121</td>
</tr>
<tr>
<td>8.0</td>
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<td>+</td>
<td>0.5000000</td>
</tr>
<tr>
<td></td>
<td>3.1415927</td>
<td>-</td>
<td></td>
<td>0.0000000</td>
</tr>
<tr>
<td></td>
<td>6.2831853</td>
<td>+</td>
<td></td>
<td>0.5000000</td>
</tr>
<tr>
<td>10.0</td>
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<td>0.0000000</td>
<td>+</td>
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<td>6.2831853</td>
<td>+</td>
<td></td>
<td>0.5000000</td>
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<tr>
<td></td>
<td>9.4247780</td>
<td>-</td>
<td></td>
<td>0.5000000</td>
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</table>

rapid maneuver, which is characterized by high fuel consumption. This maneuver requires four impulses of different magnitudes. Three impulses of different magnitudes are required for $\tau_f = 6$. The optimal fuel required for this case is close to the minimum achieved for $\tau_f \geq 2\pi$. In contrast, only two impulses are required when $\tau_f = 8$, with the optimal fuel consumption equal to the minimum possible consumption. Further increases in $\tau_f$ offer no advantage in fuel consumption for this case. However, for large $\tau_f$ there may exist multiple solutions to the selection of impulse coefficients, enabling flexibility in the optimal control system design. For example, an infinite number of pulsing schemes are possible for the case $\tau_f = 10$. The coefficients contained in Table 1 were chosen such that the control requires only one-sided pulsing. This control is demonstrated in Fig. 3. A pulse applied at $\tau = \pi$ yields a step change in $\dot{\psi}$, exciting the precessional motion of the body. A second pulse applied at $\tau = 3\pi$ terminates the precessional motion and completes the desired reorientation maneuver.

IV. Conclusion

A recently developed numerical technique for exactly solving fuel optimal control problems has been demonstrated on a classical problem. The adaptive grid bisection search was discussed and applied to the problem of fuel optimal reorienta-

tion of axisymmetric spin-stabilized rigid satellites. This approach was shown to be useful in determining the fuel optimal impulsive control strategy, particularly when rapid maneuvers are desired. For large enough maneuver times, the method produced the familiar two-impulse reorientation maneuver. For even larger maneuver times, the existence of multiple optimal pulsing schemes was revealed.

Acknowledgments

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References


Fuel Optimal Reorientation of Axisymmetric Spin-Stabilized Satellites

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I. Introduction

The problem of fuel optimal reorientation of axisymmetric spin-stabilized rigid satellites using reaction control jets was widely investigated in the 1960s. Adopting various approaches, researchers concluded that fuel optimal reorientation requires two impulse torques when the maneuver time is large enough. However, such two-impulse schemes become invalid for small maneuver times. This note applies a new numerical technique to solve the fuel optimal spin axis reorientation problem for large and small maneuver times.

For simplicity, early investigations of reorientation maneuvers employed impulse torques with no implication of fuel optimality. Later, the two-impulse scheme was shown to be fuel optimal for maneuver times that are large enough to accommodate the required precessional motion. Next, Yin and Grimmel considered the bounded input problem, showed that the optimal control was bang off bang, and developed the associated switching curves. Furthermore, it was shown that reducing the maneuver time decreases the control off time until the fuel optimal control converges to the time optimal bang-bang control. Later, the bounded fuel optimal problem was examined in detail by Wingate, who determined reorientation schemes for various geometries by numerically minimizing the associated Hamiltonian. Recently, a new technique was developed for exactly solving unbounded fuel optimal control problems for arbitrary maneuver times.

Extending the work of Krasovskii, Neustadt developed a geometric approach to solving optimal control problems. This approach was applied to fuel optimal control and resulted in a control scheme that was demonstrated on second-order systems. Recently, the development of an adaptive grid bisection search has made it possible to exactly solve optimal control problems for higher order systems with a prescribed maneuver time. This method is well suited for the rigid satellite problem, especially when rapid reorientation is desired.

The formulation of the fuel optimal reorientation of a spin-stabilized axisymmetric rigid satellite is presented in Sec. II. Nondimensional plots of fuel consumption vs maneuver time for various geometries are given in Sec. III, and some typical maneuvers are detailed. Finally, some concluding remarks are given in Sec. IV.

II. Fuel Optimal Reorientation

A rigid satellite is shown in Fig. 1. The inertial coordinates \( i_1, i_2, \) and \( i_3 \) and the body-fixed coordinates \( b_1, b_2, \) and \( b_3 \) are related by successive rotations through an appropriate selection of Euler angles. The coordinate system is first rotated about the \( i_3 \) axis through the angle \( \psi \). Next, the system is rotated about the nonspinning body \( y \) axis through the angle \( \theta \). Finally, the system is rotated about the body axis of symmetry \( b_1 \) through the angle \( \phi \). The satellite is spinning at the rate \( \Omega \) about the axis of symmetry \( b_1 \) that has an associated mass moment of inertia \( I_1 \). The transverse mass moment of inertia is denoted by \( I_2 \). The satellite is reoriented by means of body-fixed reaction control jets producing bidirectional control moments about the \( b_3 \) axis. Assuming that the control moment is initially aligned with the nonspinning body \( z \) axis, the linearized equations of motion are

\[
\begin{align*}
\frac{d^2 \psi(\tau)}{d\tau^2} + \frac{r}{\Omega} \frac{d\phi(\tau)}{d\tau} &= \cos \tau u(\tau) \\
\frac{d^2 \theta(\tau)}{d\tau^2} + \frac{\tau}{\Omega} \frac{d\psi(\tau)}{d\tau} &= \sin \tau u(\tau)
\end{align*}
\]

where \( \tau = \Omega \tau \) is the nondimensional time, \( u(\tau) = M(\tau)/I, \Omega^2 \) is the admissible nondimensional control moment, and \( r = I_r/I \) is the inertia ratio. The equation of motion was linearized assuming that \( \theta \) and \( d\psi/d\tau \) are small. Note that \( \tau \) ranges from 0 (long thin rod) to 2 (flat disk).

We define the state vector \( x(\tau) = [\psi(\tau), \phi(\tau), \theta(\tau), \theta(\tau)]^T \) and convert Eq. (1) to the first-order form

\[
x(\tau) = Ax(\tau) + b(\tau)u(\tau)
\]

where

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & r \\
0 & 0 & 0 & 1 \\
0 & -r & 0 & 0
\end{bmatrix}
\]

\[
b(\tau) = \begin{bmatrix}
0 \\
\cos \tau \\
0 \\
-\sin \tau
\end{bmatrix}
\]

The objective of the control is to transfer the system from some initial state \( x_0 \) to a final state \( x_f \) in maneuver time \( \tau_f \) while minimizing the fuel

\[
fuel = \int_0^{\tau_f} |u(\tau)| d\tau
\]

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