spacecraft to follow optimal profiles. Without lag compensation, it was verified that the fuel to carry out maneuvers increased with decreasing decay rates as a result of the increase in lag between the spacecraft and its shadow. However, with the introduction of lag compensation, the associated fuel decreased with decreasing decay rates, as is the case in vibration-suppression problems. This indicates that the lag compensating decentralized feedback maneuvers are near-optimal. Furthermore, a high level of system level reliability was achieved by suppressing the relative motion between the actual spacecraft and its shadow using decentralized feedback in which the relative motion must be measured in inertial coordinates. As an illustration, a spacecraft maneuver was successfully performed in the presence of both a shut-down actuator failure and an explosive actuator failure.

References

successfully spins up the beam, suppresses the induced vibration, and eliminates the drift of the mass center.

**D. Lag Compensation**

In Sec. III.B, the shadow coordinates followed an optimal profile throughout the maneuver. We also note that the angular velocity of the beam lagged the optimal angular velocity and that the lag decreased as the control damping rate increased. Here, we compensate for the lag by computing shadow coordinates that allow the beam to follow the optimal profile. Toward this end, using the equations of motion (8) and the control law [Eq. (11)], and assuming small angles between the floating and shadow coordinates, we obtain the closed-loop equations describing rotations about \( \dot{b}_3 \) as

\[
\dot{\Omega}_3 = 2\ddot{\ell}_c (\ddot{\Omega}_3 - \dot{\Omega}_3) - \ddot{\ell}_c (\theta_3 - \theta_{33})
\]  

(15)

in which

\[
\dot{\Omega}_3 = \theta_3, \quad \dot{\Omega}_{33} = \dot{\theta}_3
\]

with

\[\ell_c = \left( \sum_{i=1}^{6} m_i \dot{x}_i^2 \right) / I_1 \] (nondimensional control moment of inertia)

Letting \( \bar{\Omega}_3 \) be identical to the optimal profile [Eq. (13)], carrying out a differentiation and an integration to obtain \( \dot{\bar{\Omega}}_3 \) and \( \theta_3 \), respectively, and substituting the result into Eq. (15), we obtain the angular velocity of the shadow coordinates with lag compensation

\[
\bar{\Omega}_{33}(t) = \frac{\dot{\Omega}_m}{T_m^2} (2T_m - \bar{t}) - \frac{2\Omega_m}{\bar{c}^2 T_m^2} \bar{c}^2 T_m e^{-\bar{c}^2/2} \] 

\[
\bar{\Omega}_1 = 0, \quad \bar{\Omega}_2 = 0, \quad \bar{t} \leq T_m; \quad \bar{\Omega}_{33}(t) = \bar{\Omega}_m, \quad \bar{t} > T_m
\]  

(16)

Figure 9 shows the desired angular velocity of the beam \( \dot{\Omega}_3 \) [Eq. (13)], the actual beam angular velocity \( \bar{\Omega}_3 \), and the shadow beam angular velocity \( \bar{\Omega}_{33} \) [Eq. (16)]. As shown, the actual beam closely follows the optimal profile with lag compensation. The difference between the angular velocity of the beam \( \bar{\Omega}_3 \) and the desired angular velocity \( \bar{\Omega}_{33} \) [Eq. (13)] is due to a small angle approximation made between the floating and shadow coordinates [Eq. (15)]. Figure 10 shows changes in fuel with respect to the maneuver time for several damping rates. Note that the fuel decreases with the introduction of lag compensation since the optimal profile is more closely followed.

**IV. Decentralized Feedback Rest-to-Rest Maneuvers**

**A. In-Plane 180-deg Maneuver**

We select shadow coordinates identical to the body-fixed coordinates associated with a minimum fuel 180-deg rest-to-rest maneuver of a rigid body with lag compensation. With the introduction of the lag compensation, the beam undergoes the indicated minimum fuel maneuver. Assuming a small angle approximation between the floating and shadow coordinates, the angular velocities of the shadow coordinates with lag compensation become

\[
\bar{\Omega}_{33} = \frac{6\pi}{T_m^2} (T_m - \bar{t}) + \left\{ -\frac{\pi}{3c^2 T_m} \right\}
\]

\[
\bar{\Omega}_1 = \bar{\Omega}_2 = 0, \quad \bar{t} \leq T_m, \quad \theta_{33} = \pi, \quad \bar{\Omega}_{33} = 0, \quad \bar{t} > T_m
\]  

(17)

Figure 11 shows the fuel vs. maneuver time for in-plane 180-deg rest-to-rest maneuvers using an optimal profile for the shadow beam.

**Fig. 9** Angular velocities for spin-up maneuvers using a lag compensating shadow beam [the desired angular velocity of the actual beam; using Eq. (13)].

**Fig. 10** Fuel vs maneuver time for spin-up maneuvers using a lag compensating shadow beam.

**Fig. 12** Time lapsed of the beam's right half with the elastic deflections magnified by 15 and with one-tenth the fundamental period time lapsed.
where

\[ F_r = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} \mathbf{j} \, dx = \sum_{r=1}^{n} \mathbf{j}_{BR}, \quad Q_r = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} \mathbf{\phi}_{BR} \mathbf{j} \, dx \]  \hspace{1cm} (9)

and we obtain the nondimensional eigenvalue problem

\[ \frac{d^2 \mathbf{\phi}_{BR}}{dx^2} = \alpha^2 \mathbf{\phi}_{BR} \]  \hspace{1cm} (10)

in which primes represent derivatives with respect to nondimensional time with

\[ \bar{u}_{01} = u_{01}/I, \quad \bar{u}_{02} = u_{02}/I, \quad \bar{u}_{03} = u_{03}/I \]  \hspace{1cm} (nondimensional positions of beam mass center)

\[ F_1 = F_1 T^2/Ml, \quad F_2 = F_2 T^2/Ml, \quad F_3 = F_3 T^2/Ml \]  \hspace{1cm} (nondimensional net external forces acting on the beam)

\[ \bar{M}_1 = M_1 T^2/I_1, \quad \bar{M}_2 = M_2 T^2/I_2, \quad \bar{M}_3 = M_3 T^2/I_3 \]  \hspace{1cm} (nondimensional net external moment about the beam mass center)

\[ \bar{q}_r = q_r/M^{\frac{3}{2}} \]  \hspace{1cm} (rth nondimensional modal displacement)

\[ \bar{\mathbf{j}} = f T^2/M \]  \hspace{1cm} (nondimensional force per unit length of the beam)

\[ \bar{\mathbf{\phi}}_{BR} = \mathbf{\phi}_{BR} M^{\frac{1}{2}} \]  \hspace{1cm} (rth nondimensional natural mode of vibration)

\[ \bar{\omega}_r = \omega_r T \]  \hspace{1cm} (rth nondimensional natural frequency of oscillation (the fundamental nondimensional frequency \( \bar{\omega}_1 = \bar{\omega}_1 = 2\pi \))

\[ \bar{j}_r = j_r T^2/Ml \]  \hspace{1cm} (rth nondimensional discrete control force in the \( \bar{b}_r \) direction located at \( \bar{x}_r = x_r/l \))

and with

\[ \alpha_1 = \frac{I_1 - I_3}{I_1}, \quad \alpha_2 = \frac{I_2 - I_1}{I_2}, \quad \alpha_3 = \frac{I_1 - I_2}{I_3} = -1 \]

Finally, the nondimensional control law has the form

\[ \bar{j}_r = -2\alpha m_r (\bar{u}_{r1} - \bar{u}_{r0}) - \bar{\alpha} \bar{\omega}_r m_r (\bar{u}_{r0} - \bar{u}_{r0}) \]  \hspace{1cm} (11a)

\[ \bar{M}_1 = -2\alpha (\theta_1 - \theta_0) - \bar{\alpha}^2 (\theta_1 - \theta_0) \]  \hspace{1cm} (11b)

with

\[ \alpha = \alpha T \]  \hspace{1cm} (nondimensional designer chosen uniform decay rate)

\[ m_r = m_r/M \]  \hspace{1cm} (nondimensional mass in the subdomain of the rth control force (\( m_1 = m_6 = 0.1, m_2 = m_3 = m_4 = m_5 = 0.2 \))]

\[ \bar{u}_{r0} = u_{r0}/l \]  \hspace{1cm} (nondimensional inertial position of point \( \bar{x}_r = x_r/l \) in the \( \bar{b}_r \) direction)

\[ \bar{u}_{r0} = u_{r0}/l \]  \hspace{1cm} (nondimensional inertial position of point \( \bar{x}_r \) in the \( \bar{b}_r \) direction on the shadow beam)

Throughout the remainder of the paper, the elastic motion of the beam is expressed as a linear combination of the 10 lowest natural modes of vibration. The contribution of the remaining modes to the overall system response is neglected.

### III. Decentralized Feedback Spin-up Maneuvers

#### A. Angular Velocity of Shadow Coordinates Chosen as Step Function

As mentioned previously, decentralized feedback maneuver requires the selection of an appropriate shadow spacecraft relative to which spacecraft motion is suppressed. As a first illustration, consider shadow coordinates rotating about the principal axis \( \bar{b}_3 \) of the beam at an angular velocity in the form of the step function.

\[ \bar{\Omega}_{b3}(t) = 0, \quad \bar{\Omega}_{b2}(t) = 0, \quad \bar{\Omega}_{b1}(t) = \bar{\Omega}_{m} \bar{u}(t) \]  \hspace{1cm} (12)

with

\[ \bar{\Omega}_{m} = \text{nondimensional angular velocity of the shadow beam about } \bar{b}_3 \]

\[ \bar{u}(t) = \text{unit step function} \]

In broad terms, the nondimensional angular velocity of the beam in the \( \bar{b}_3 \) direction, \( \bar{\Omega}_{b3} \), is expected to lag the nondimensional angular velocity of the shadow coordinates \( \bar{\Omega}_{b3} \) increasing as the nondimensional decay rate \( \bar{\alpha} \) decreases. As a rule of thumb, \( \bar{\alpha} \) must be chosen to be at least one-third the fundamental frequency of the beam. Otherwise, the control law does not allow the beam to “catch up” to the shadow beam. It follows that both the rise time and the settling time associated with \( \bar{\Omega}_{b3} \), decrease as \( \bar{\alpha} \) increases, as shown in Fig. 3.

#### B. Angular Velocity of Shadow Coordinates Chosen as Optimal Profile

Previously, the angular velocity of the shadow coordinates were chosen in the form of a step function [Eqs. (12)]. In this

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**Fig. 3** Rise time and settling time of \( \bar{\Omega}_{b3} \) for spin-up maneuvering using a step function as the angular velocity for the shadow beam.
Decentralized Feedback Maneuver of Flexible Spacecraft

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This paper describes a novel approach to flexible spacecraft maneuver in which spacecraft motion relative to the motion of a shadow spacecraft is suppressed using a decentralized feedback control. The shadow spacecraft is a fictitious spacecraft that can be positioned, oriented, and allowed to undergo elastic deformations at the discretion of the designer. First, rest-to-spin maneuvers are investigated, with different choices for the shadow coordinates. The angular velocities of the shadow coordinates are chosen as step functions, quadratic functions corresponding to minimum fuel maneuvers for rigid bodies, and quadratic functions with lag compensation in which the lag is associated with the spacecraft angular velocities relative to the shadow spacecraft angular velocities. The performance of the maneuvers is illustrated in the presence of multiple actuator failures. Next, both in-plane and out-of-plane rest-to-rest maneuvers are investigated. The associated shadow coordinates are chosen to those of minimum fuel maneuvers and minimum fuel maneuvers with lag compensation.

I. Introduction

The problem of maneuvering flexible spacecraft has traditionally been cast in the form of nonlinear two-point boundary-value problems. Although solutions to the general nonlinear two-point boundary-value problem can be obtained for rigid spacecraft, the computational effort and the numerical sensitivities can often pose problems. For flexible spacecraft, the solution of the nonlinear two-point boundary-value problem may be close to unattainable in the extreme case and, at the very least, provides little insight into algorithms well suited for in-space implementation. This paper describes a novel approach to the problem of maneuvering flexible spacecraft in which spacecraft motion relative to the motion of a shadow spacecraft is suppressed using a decentralized feedback control. The shadow spacecraft is a fictitious spacecraft whose motion is freely prescribed by the designer. Thus, the shadow spacecraft may be positioned and oriented and may undergo flexible-body motions if so desired. An inherent level of system reliability is then obtained by suppressing motion relative to the shadow spacecraft using decentralized feedback.

In Sec. II, the dynamics of a nondimensional free-free beam with well-separated dimensions are described, and a general formulation for the associated control law is given. This representative spacecraft will be used throughout the remainder of the paper. In Sec. III, we investigate decentralized feedback spin-up maneuvers with different choices for the shadow spacecraft. Decentralized feedback rest-to-rest maneuvers are then considered in Sec. IV.

II. Maneuver of a Free-Free Beam with Well-Separated Dimensions

Consider the free-free uniform beam of length \( L \), width \( w \), and cross-sectional thickness \( t \), with well-separated dimensions so that \( c \ll w \ll L \). The beam undergoes bending vibration in the \( z \) direction (see Fig. 1), and so the stiffness operator for the beam has the form \( E = EI^2 / 3 x^4 b z^2 \), where \( EI \) denotes the bending stiffness of the beam. The mass per unit length of the beam is denoted by \( \rho \), with \( M = \rho L \) representing the total mass of the beam and \( I = (M/12)(w^2 + c^2) \), \( I_2 = (M/12)(w^2 + l^2) \), and \( I_3 = (M/12)(c^2 + l^2) \) representing the principal mass moments of inertia of the beam about the three axes of the floating coordinates, respectively. The initial position of any point on the spacecraft is expressed as

\[
u = u_0 + u_C + u_E\]

with

\[
u_0 = u_0(t) = u_0(t) \hat{r}_1 + u_0(t) \hat{r}_2 + u_0(t) \hat{r}_3\]


which is the position of the beam's mass center (coincident with the origin of the floating coordinates) measured in inertial coordinates with unit vectors \( \hat{r}_1 \), \( \hat{r}_2 \), and \( \hat{r}_3 \), and

\[
u_C = u_C(P, t) = x_1 \hat{b}_1(t) + x_2 \hat{b}_2(t) + x_3 \hat{b}_3(t)\]

which is the nominal position to point \( P \) relative to the mass center of the beam measured in floating coordinates with unit vectors \( \hat{b}_1(t) \), \( \hat{b}_2(t) \), and \( \hat{b}_3(t) \) (see Fig. 2).

In the case of our free-free beam, \( u_C(P) = x \hat{b}_2 \). The elastic displacement is orthogonal to the rigid-body modes of vibration and expressed as a linear combination of elastic modes of vibration measured relative to the floating coordinates as

\[
u_E = \sum_{r=1}^{n} \phi_{ER} \hat{a_r}\]

with \( u_E = u_E(P, t) = u_{ER}(P, t) \hat{a}_1 + u_{ER}(P, t) \hat{a}_2 + u_{ER}(P, t) \hat{a}_3 \), the elastic displacement of the spacecraft; \( \phi_{ER} = \phi_{ER}(P) = \phi_{ER}(P) \hat{a}_1 + \phi_{ER}(P) \hat{a}_2 + \phi_{ER}(P) \hat{a}_3 \), the \( r \)th elastic mode of

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