Space-Based Electrostatic Antenna Design with Pointing and Beam Width Control

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This paper describes a new type of adaptable antenna that points over a 6-deg range and whose directivity is varied over a 6.5-dB range in experimental tests. The electrostatic antenna concept presented here can be regarded as a compromise between the low-performance mechanical method of adapting the far-field beam pattern and the high-performance but often prohibitively complex method of carrying out the adaptations electronically. The electrostatic antenna described in this paper is shown to be well suited for space-based applications.

Nomenclature

\[ a, b, c, d, e, f, g \]
= undetermined coefficients

\[ a \]
= vector of undetermined coefficients

\[ a_1, a_2 \]
= vectors of undetermined coefficients

\[ f \]
= focal length

\[ M \]
= number of test cases

\[ r, s, i \]
= counting indices

\[ T \]
= time constant

\[ V \]
= voltage

\[ V_1 \]
= applied voltage pair left of center

\[ V_2 \]
= applied voltage pair right of center

\[ X, Y \]
= coefficient matrices

\[ x \]
= abscissa

\[ y \]
= surface deflection

\[ y \]
= vector of displacements

\[ \Psi_0 \]
= desired surface deflection

\[ \Delta V_1 \]
= resolution of \( \Psi_0 \)

\[ \Delta V_2 \]
= resolution of \( \Psi_0 \)

\[ \Delta y \]
= displacement error

\[ \delta V \]
= difference between voltage pairs \( V_1 \) and \( V_2 \); \( V_1 - V_2 \)

\[ \theta \]
= angle between translated and rotated axis

\[ \lambda \]
= signal wavelength

Introduction

The concept of harnessing electrostatic forces for membrane shaping surfaced again in the early 1990s at North Carolina State University. The first author, along with a team of graduate students, independent of the a priori work, envisioned an electrostatically shaped membrane that differed from Perkin's electrostatic membrane in one important feature. Silverberg's electrostatically shaped membrane contained no supporting structure; the outside edges were left unconstrained. This concept variation was significant enough to warrant a new patent in April of 1994. The research was carried out in the following progression: They were first concerned with how reliable one could predict electrostatically induced deformations. The next step was a general formulation for predicting the dynamic behavior of structures subjected to electrostatic fields. Once the dynamics of electrostatic structures were understood, the effort turned toward investigating control issues. One study developed a modal control technique for shaping the antenna surface to maximize the far-field power density.

This paper focuses on the basic principles that govern the overall design of electrostatic antennas. The next section describes the performance characteristics that are achievable with the electrostatic antenna described in this paper and how these differ from a fixed shape antenna. In particular, we review the abilities of the electrostatic antenna to control pointing, directivity, and, more generally, the beam-pattern distribution. The third section provides detailed descriptions of the experimental setup, covering geometry, the membrane, power requirements, type of feed, pinned boundary conditions, back plates, measurement system, and a photo of the experiment. In the fourth section, empirical relationships that collectively govern antenna performance are developed. Surface

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Fig. 1. Goslee's electrostatic antenna (16 ft in diameter); modified back plates. [Image: Figure 1 showing modified back plates for antennae, labeled E and F.]
deflections are expressed as a function of applied voltages, and the far-field beam pattern of the transmitted antenna signal is calculated. Voltage resolution and surface accuracy are then expressed as functions of the transmission wavelength. The fifth section gives experimental results that demonstrate the performance characteristics of the electrostatic antenna. This section shows how well the antenna beam can be pointed, its directivity varied, and its surface resolution maintained, in addition to other engineering performance characteristics.

Antenna Performance

The earliest human-made far-field antennas date back to the 1920s with the ability to transmit and receive short wavelength signals for radio detection. Monopole and dipole antennas were first used and later directional antennas were designed to yield desirable beam patterns. The next level of sophistication in antenna design arose with the introduction of adaptable antennas wherein the beam pattern is adapted in real time. Adaptable antennas are currently one of two types: mechanical or electronic (phased array). This paper describes another type of adaptable antenna, called an electrostatic antenna, that is particularly well suited for space-based applications.

Unlike the mechanical and electronic types, the electrostatic antenna's beam pattern is adapted by varying the shape of the antenna's surface, specifically, its modes of deformation. The first mode of deformation is called the pointing mode. This mode is responsible for global beam pointing. The pointing mode enables the antenna to scan and to function for surveillance and mapping purposes. The second mode of deformation controls the beam width. In space applications, the beam width mode controls the ground swath illuminated on the Earth by the antenna. The beam width mode is useful in communication satellites and in certain military applications. Higher modes of deformation control the general power distribution of the beam.

Within the context of space-based adaptable antennas, the mechanical types tend to suffer from the following limitations. Their beam pattern adaptability is restricted to pointing (mode one deformation), and the associated beam pattern time constants are relatively large ($T > 500$ s). On the other hand, the electronic types can control the beam pattern rapidly ($T < 1$ ms), but heat rejection problems and other manufacturing complexities have inhibited their use. The electrostatic antenna, as this paper demonstrates, is capable of controlling the first two modes of the beam pattern (pointing and beam width). The control of the higher modes has not yet been demonstrated, although it is feasible, in principle. The associated beam pattern time constants are smaller than for the mechanical type and larger than for the electronic type. The relatively small beam pattern time constants associated with electrostatic antenna ($T < 10$ s) are achievable because the aperture is composed of a low-mass membrane that is weakly coupled to the satellite dynamics.

Setup

An electrostatic antenna works on the basic principle that like charges repel. When two electrodes are charged, with one fixed and one free to move, the free electrode will move apart from the fixed electrode so as to balance the elastic restoring force and the electrostatic force. By controlling the applied voltages, the position of the free electrode can be prescribed. On this basis, an electrostatic antenna is designed here to control its first two modes of deformation, which in turn control pointing and beam width (see Fig. 2).

In the following experiment, the fixed electrodes (see 2 and 4 in Fig. 3) are referred to as back plates, and the free electrodes (see 3 and 5 in Fig. 3) are electrically segmented regions on a 7-mil metallized mylar membrane. Metallized mylar was chosen because it is elastic, conductive, and easily segmented. Metal between segments was removed using a liquid solution of sodium hydroxide (NaOH) (see Fig. 4).

The membrane is physically constrained by a magnetically levitated pinned support (see 6 in Fig. 3). The support is composed of two semicylinders that form a cylinder when brought together (1-in. diam, 13-in. length). Seven pairs of attracting magnets secure the centerline of the membrane along the constrained interaxial nating regions of insulative and conductive material (see 7 and 8 in Fig. 5, respectively). These regions provide a conductive pathway along which the membrane segments are charged. The support is constrained in the horizontal plane by high-precision ball bearings (see 9 in Fig. 5). In the vertical direction the support is levitated by neodymium-iron-boron magnets. This combination produces a near frictionless pinned boundary condition at the center of the membrane while allowing the edges to freely deflect.

The back plates are composed of thin aluminum rectangular plates; however, just about any conductive material would work. The number of back plates is governed by the number of controlled modes of deformation, in this experiment two. As a general rule, the number of independently controlled back plates is not less than the number of controlled modes of deformation. In optical applications, the surface tolerance requirements may not be met with the minimum number of back plates, in which case more back plates are used until the desired surface accuracy is reached. In microwave applications, the number of controlled back plates is on the order of the number of controlled modes of deformation. Since we seek to control two modes, we use two back plates, each paired with one side of the membrane. Also note that the fixed back plates may be eliminated in some designs by instead using attached back plates.4

The back plates along with the support are mounted on a multipurpose high-voltage test block. The test block consists of nothing more than an 18 x 12 x 4-in. block of acrylic with uniformly spaced holes that accept spring loaded RJ-8 coaxial cables from 0-30 kV dc power supplies. The springs act as motor brushes, which allow the two parts of the membrane to be charged while not inhibiting the motion.
Fig. 4 Electrically segmented metalized mylar: 20) left side of electrically segmented metalized mylar and 21) right side of electrically segmented metalized mylar.

Fig. 5 Magnetically levitated pinned support: 7) insulative material, 8) conductive material, 9) high-precision ball bearing and magnet well, 10) high-voltage test block mounting slots, and 11) attracting magnets.

Table 1 Applied voltages for each scan (M = 11)

<table>
<thead>
<tr>
<th>$V_1$ (kV)</th>
<th>$V_2$ (kV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>10</td>
<td>X</td>
</tr>
<tr>
<td>15</td>
<td>X X X X X</td>
</tr>
<tr>
<td>20</td>
<td>X X X X X</td>
</tr>
</tbody>
</table>

resolution of 0.034 in. The analog distance sensor rides broadside to the aperture on the linear positioner via a carriage. By recording the 0–5 V analog output signal from the sensor at specified intervals, the antenna surface is recorded (see Fig. 6).

Engineering Curves

The engineering performance of the electrostatic antenna is described by relationships (curves) between the applied voltages, the surface deformations, and the beam pattern. First, the surface deformations are expressed as a function of the applied voltages. Next, beam pattern parameters are expressed as functions of the surface deformations. Finally, surface rms values, and resolution issues are discussed.

The deformation of a surface cross section is assumed in the polynomial form

$$y(x, V_1, V_2) = \sum_{r=0}^{4} a_r(V_1, V_2)x^r, \quad x_1 \leq x \leq x_N \quad (1)$$

where

$$a_r(V_1, V_2) = b_r + V_1c_r + V_2d_r + V_1^2e_r + V_2^2f_r + V_1V_2g_r \quad (2)$$

The previously described measurement system scans the membrane and records surface deformations at $x_s (s = 1, 2, \ldots, N)$. The measurement system scans the membrane $M = 11$ distinct times; each time the applied voltages $V_1$ and $V_2$ are updated (see Table 1). Deformation curves are obtained from the data in two steps. The first step determines $a_r (r = 0, 1, \ldots, 4)$ in Eq. (1) for each scan. The second step determines $b_r, c_r, d_r, f_r,$ and $g_r (r = 0, 1, \ldots, 4)$ as a function of $V_1$ and $V_2$. For each scan, the first step yields from Eq. (1) the set of linear algebraic equations

$$Xa = y \quad (3)$$

in which $a = [a_0, a_1, \ldots, a_4]^T, y = [y(x_1), y(x_2), \ldots, y(x_N)]^T$, and

$$X = \begin{bmatrix}
1 & x_1 & x_1^2 & x_1^3 & x_1^4 \\
1 & x_2 & x_2^2 & x_2^3 & x_2^4 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & x_N & x_N^2 & x_N^3 & x_N^4
\end{bmatrix}.$$
The least squares estimate of \( a \) is given by
\[
a = (X^T X)^{-1} X^T y
\]

(4)

The second step yields from Eqs. (2) and (4) the set of linear algebraic equations
\[
V_{cr} = a_r, \quad (r = 0, 1, \ldots, 4)
\]

(5)
in which \( a_r = [a_r(1) \quad a_r(2) \ldots \quad a_r(M)]^T \), \( z_r = [b, c, d, \ldots, f, g, \ldots]^T \), and
\[
V = \begin{bmatrix}
1 & V_1(1) & V_2(1) & V_3^2(1) & V_4^2(1) & V_5(1)V_2(1) \\
1 & V_1(2) & V_2(2) & V_3^2(2) & V_4^2(2) & V_5(2)V_2(2) \\
1 & V_1(3) & V_2(3) & V_3^2(3) & V_4^2(3) & V_5(3)V_2(3) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & V_1(M) & V_2(M) & V_3^2(M) & V_4^2(M) & V_5(M)V_2(M)
\end{bmatrix}
\]

Finally, the least squares estimates of \( z_r \) \((r = 0, 1, \ldots, 4)\) are given by
\[
z_r = (V^T V)^{-1} V^T a_r, \quad (r = 0, 1, \ldots, 4)
\]

(6)

The surface deformations described earlier were obtained empirically rather than predicted from a physical model of the system. The prediction of surface deformations of electrostatic structures is possible but was unnecessary in this investigation.\(^6\)

Having expressed deformations as functions of the applied voltages, the next step is to determine the far-field beam pattern and its associated performance parameters (pointing, beam width, etc.) as functions of the surface deformations. Rather than perform this part of the investigation experimentally, we turned to a very accurate method of prediction that is based on physical optics. Many commercially available computer subroutines exist; in our study we used POMESH.\(^9\)

Another useful set of parameter curves governs surface deformation sensitivity. The rms value of the difference between the actual surface deformation and the desired surface deformation is determined in several steps. The surface deformations are first translated to a set of axes whose origin lies on the vertex of the membrane. A weighted average of the pointing of the surface deformations are then determined by
\[
\theta = \frac{1}{2} \frac{\sum_{r=1}^{N/2} [y(x_r) - y(-x_r)] x_r}{\sum_{r=1}^{N/2} x_r^2} = \frac{\sum_{r=1}^{N/2} \theta_i x_i^2}{\sum_{i=1}^{N/2} x_i^2}
\]

(7)
in which \( \theta_i = \frac{[y(x_i) - y(-x_i) / 2x_i \quad \text{denotes the pointing angle associated with } x_i \quad \text{and } x_i \quad (x_i > 0)}{\text{Note that } \theta \text{ also represents the least squares solution}} \text{ obtained by minimizing the error function}} \]

\[
E = \sum_{i=1}^{N/2} [y(x_i) - y(-x_i) - 2\theta x_i]^2
\]

\[
= \sum_{i=1}^{N/2} [(y(x_i) - \theta x_i) - (y(-x_i) + \theta x_i)]^2
\]

The translated coordinates are now rotated an amount \( \theta \). The desired surface deformations in the rotated frame are given by the general parabolic form \( y_d = (1/4f)x^2 \), and the associated rms value of the difference between the actual surface deformation and the desired surface deformation is given by

\[
\text{rms} = \left\{ \frac{1}{N} \sum_{r=1}^{N} \left[ V_r - V_d(x_r, V_1, V_2) \right]^2 \right\}^{1/2}
\]

(8)
in which \( V_r(x_r, V_1, V_2) \) denotes the desired surface deformation as a function of \( x_r, V_1, \) and \( V_2 \) in the rotated coordinate system. Another rms value of importance is associated with the error introduced by the limiting resolution of the applied voltages \( V_1 \) and \( V_2 \).

We obtain

\[
\text{rms} = \left\{ \frac{1}{N} \sum_{r=1}^{N} \left[ \frac{\partial y(x_r, V_1, V_2)}{\partial V_1} \Delta V_1 + \frac{\partial y(x_r, V_1, V_2)}{\partial V_2} \Delta V_2 \right]^2 \right\}^{1/2}
\]

(9)
in which the inner parentheses term denotes the displacement error. Both rms errors given in Eqs. (8) and (9) contribute independently to the total rms error

\[
\text{rms} = \text{rms}_1 + \text{rms}_2
\]

(10)
The transmitted or received signal that coherently reflects off of the aperture bounds the total rms by

\[
\text{rms} < \lambda/10
\]

(11)
Equation (1) determines the surface deformations as functions of the applied voltages, which in turn are related to the beam pattern, as previously described, and the surface deformations are related to acceptable rms levels as indicated by Eq. (11).

**Performance Tests**

The electrostatic antenna described in this paper was designed to undergo the first two modes of deformation: the pointing mode and the beam width mode. Upon completing \( M = 11 \) independent scans, the antenna's deformation as a function of position and applied voltage pairs \( V_1 \) and \( V_2 \) [Eqs. (1) and (2)] were computed. The results are given in Table 2. The simulated far-field patterns used a transmission frequency of 1 GHz (\( \lambda = 18.80 \text{ in.} \)) and a dipole feed. We first evaluate the pointing mode. As shown in Fig. 7 and referring

![Fig. 7 Pointing mode.](image_url)

<table>
<thead>
<tr>
<th>Table 2 Deflection coefficients vs ( r = 0, 1, 2, 3, 4 )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>1.3449e+01</td>
<td>3.0670e-01</td>
<td>-4.7906e-02</td>
<td>3.8650e-03</td>
<td>-1.4097e-04</td>
</tr>
<tr>
<td>( c )</td>
<td>-1.3158e-02</td>
<td>4.1077e-03</td>
<td>-7.8152e-05</td>
<td>5.6697e-06</td>
<td>3.7078e-06</td>
</tr>
<tr>
<td>( d )</td>
<td>-1.9472e-03</td>
<td>7.6749e-04</td>
<td>-5.4300e-05</td>
<td>9.4435e-06</td>
<td>-4.6386e-06</td>
</tr>
</tbody>
</table>
to Table 1, the rotation of the antenna is proportional to the difference between the applied voltage to the left of center ($V_1$) and the applied voltage to the right of center ($V_2$), denoted by $\delta V = V_1 - V_2$. From Fig. 7 the antenna rotates approximately 1 deg every $\delta V = 5$ kV. The associated beam pattern is shown in Fig. 8. As shown, the pointing also increases approximately 1 deg every $\delta V = 5$ kV. This corresponds to the best parabolic fit to the uncharged preshaped membrane ($V_1 = V_2 = 0$). The desired antenna surface was assumed to be parabolic with a focal length of $f_0 = 15.84$ in. The rms values corresponding to Fig. 7 are $\text{rms}_1 = (0.47, 0.51, 0.54, 0.70, 0.76)$. Once again $\text{rms}_2 \ll \text{rms}_1$, and the beam width mode met the antenna wavelength requirements bounded below by $\lambda_{\text{min}} = 7.6$ in.

**Summary**

This paper described the design of an experimental electrostatic antenna and tested its performance. The experimental antenna was shown to be capable of adapting the pointing of the main beam over a 1-deg range and the beam width over a 5-deg range. The experimental antenna surface was sufficiently accurate to accommodate wavelengths bounded below by approximately 7.6 in.

Future efforts could extend the results presented here to the design and testing of electrostatic antennas that are paraboloidally preshaped and to the increase in the number of independently applied voltages along with an increase in the number of modes of deformation that are shaped.

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