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ADAPTIVE ELECTROSTATIC STRUCTURES: A FUNDAMENTAL STUDY OF THE ELECTROSTATIC OSCILLATOR

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Introduction

Electrostatic structures are a new class of adaptive structures in which the structure is imbedded with electrical charge over its domain, which in turn is controlled by an external electrical field. The external electrical field is obtained by a distribution of electrodes, each charged to a prescribed potential. The result is a structure that can change its shape.

Fundamental experiments in this area accurately predicted equilibrium positions within errors on the order of 0.16% (Ref. 1, 2) and predicted electrostatic frequencies of oscillation to within errors of 10-20%. While the prediction of the equilibrium positions is sufficiently accurate to validate the prediction techniques employed, the prediction of the electrostatic frequencies was insufficiently accurate and raised questions regarding the prediction method employed and several of the associated assumptions. This paper will describe an experiment and prediction method specifically designed to resolve the inaccuracy of the predicted natural frequency obtained in Refs. 1 and 2. In the process, this paper will highlight fundamental issues in predictive modeling of adaptive electrostatic structures.

The field of electrostatics dates back to the classical works of Coulomb and since then has been honored by many historic works. Likewise, the field of mechanics dates back just as far, and within its domain contains a great many classical works. In contrast, the coupling of the two fields is of recent vintage. Coupled problems of electrostatics and mechanics are found in contemporary engineering applications. Examples of these include electrostatic speakers (Refs. 3 and 4), scientific instruments (Refs. 5 and 6) and space-based antennas (Refs. 7 and 8). One way or another, these electrostatic dynamical systems can be regarded as complex electrostatic oscillators. This paper develops an electrostatic oscillator experiment, predicts its behavior and compares the predictions with the measurements.

The electrostatic oscillator experiment presented here will isolate the electrical forces and balance to zero the mechanical forces so that the frequency of the oscillator is
due to the electrostatic effects alone. Spherical geometries were selected for the charged surfaces so that the predictions can be made using the method of images (Ref. 9). Furthermore, the parameters were selected to yield a frequency in a range that is easily observed (and measured).

Set Up

The electrostatic oscillator consists of a long slender rod and cross-bar pivoted on two pins as shown in Fig. 1. A conductive sphere of radius $R = 1.850$ cm is fixed to each end of the rod. Another conductive sphere of identical radius lies below each sphere. The gap between each pair of spheres is $G_0 = 2.350$ cm when the rod is horizontal. The four spheres are charged to a constant voltage $V_0$.

![Figure 1. Set up](image)

The rod length $B = 37.1$ cm is large compared to the sphere radius. Therefore, the electrical forces produced on one side of the rod by the spheres on the other side of the rod are ignored, that is we ignore the electrical forces across the rod, and any effects that these forces would produce. The charges on the spheres are all positive and the electrical force across either gap decreases as the size of the gap increases. Therefore, as the rod rotates either clockwise or counter clockwise, a counteracting moment is exerted on the rod by the electrical forces across the two gaps. This establishes the oscillatory behavior of the rod.
Predictions

Electrical charge distributed over the two spheres produces electrical forces across the two gaps. The electrical force across either gap depends on the sphere radius \( R \), the gap length \( G \) and the applied voltage \( V_0 \). The electrical force is predicted here by the method of images. Other methods would be employed in problems in which the geometry is more complex, (see for example, Refs. 10 and 11). The method of images replaces the two-sphere charge distribution with an infinite series of pairs of fictitious point charges that reproduce the voltage \( V_0 \) over the surfaces of the two spheres. Referring to Fig. 2, the first pair of point charges is located at \( a_1 = 0 \) (letting \( r = 0 \)) and the value of each charge is \( Q_0 = kV_0/k \), where \( k = 9 \times 10^9 \) Nm\(^2\)C\(^{-2}\). The charge \( Q_0 \) is identical to the charge on an isolated sphere of radius \( R \) and applied voltage \( V_0 \). The \( r \)th pair of point charges is located at

\[
\begin{align*}
  a_r & = \frac{R^2}{D - a_{r-1}}, \quad (r = 1, 2, ...,) \quad (1) \\
  Q_r & = -\frac{a_r}{R} Q_{r-1}, \quad (r = 1, 2, ...) \quad (2)
\end{align*}
\]

The force across the gap becomes

\[
F(D) = k \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{Q_r Q_s}{(D - (a_r + a_s))^2} \quad (3)
\]

The voltage in free space produced by the point charges becomes

\[
V(x, \alpha) = k \sum_{r=0}^{\infty} Q_r \left( \frac{1}{D_{1r}} + \frac{1}{D_{2r}} \right) \quad (4)
\]

in which

\[
\begin{align*}
  D_{1r}^2 &= x^2 + a_r^2 - 2a_r x \cos \alpha \\
  D_{2r}^2 &= x^2 + (D - a_r)^2 - 2(D - a_r) x \cos \alpha
\end{align*}
\]

Note when we let \( x = R \) in Eq. (4) that we should obtain the applied voltage on the surface of the sphere \( V(R, \alpha) = V_0 \). The charge per unit area on the surface of the spheres can also be predicted from Eq. (4). We get (Ref. 9)

\[
\sigma(\alpha) = -\frac{1}{4\pi k} \left. \frac{\partial V(x, \alpha)}{\partial x} \right|_{x=R} = \sum_{r=0}^{\infty} \frac{R - a_r \cos \alpha}{(R^2 + a_r^2 - 2Ra_r \cos \alpha)^{3/2}}
\]

\[
+ \frac{R - (D - a_r) \cos \alpha}{(R^2 + (D - a_r)^2 - 2R(D - a_r) \cos \alpha)^{3/2}} Q_r \quad (6)
\]

The oscillatory behavior of the rod can now be established. Summing moments about the pivot point, we obtain the countering moment and the predicted natural frequency of oscillation.

\[
\omega^2 = -\frac{1}{l} \left. \frac{\partial M_0(\theta)}{\partial \theta} \right|_{\theta=0} = -\frac{B^2}{2I} \left. \frac{\partial F(D)}{\partial D} \right|_{D=D_0}
\]

\[
= -\frac{B^2 k}{2I} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{-2Q_r Q_s}{(D - (a_r + a_s))^3} \left( 1 - \frac{\partial a_r}{\partial D} \frac{\partial a_s}{\partial D} \right)
\]

\[
+ \frac{\partial Q_r}{\partial D} Q_s + \frac{\partial Q_s}{\partial D} Q_r \bigg|_{D=D_0} \quad (7)
\]

where

\[
f(\partial a_r, \partial D) = -\left( \frac{R}{D - a_r} \right)^2 \left( 1 - \frac{\partial a_{r-1}}{\partial D} \right)
\]

\[
\frac{\partial Q_r}{\partial D} = -\frac{1}{R} \left( \frac{\partial a_r}{\partial D} Q_{r-1} + a_r \frac{\partial Q_{r-1}}{\partial D} \right)
\]

in which \( l \) denotes the mass moment of inertia about the pivot point, \( \frac{\partial a_0}{\partial D} = \frac{\partial Q_0}{\partial D} = 0 \), and \( \omega \) denotes the predicted natural frequency of oscillation.
Preliminary Results

The natural frequency prediction given in Eq. (7) requires a truncation of the number of pairs of point charges. Denoting this number by \( n \), the accuracy of the approximation can be determined indirectly by checking the accuracy of the voltage at the sphere boundary. Figure 3 shows \( V(R, \alpha) \) obtained from Eq. (4) letting \( V_0 = 1,000 \) V. The natural frequency predictions versus \( n \) are shown in Fig. 4. The associated contour plot of lines of constant voltage for \( n = 10 \) is shown in Fig. 5.

The above described results are incomplete. Further results will include measured natural frequencies of oscillation, and a description of the sources of errors responsible for the discrepancies between the measured and the predicted natural frequencies.

Figure 3. Predicted voltage at the sphere boundary

Figure 4. Predicted natural frequency

In addition, a comparison between predicted and measured equilibrium angles will be made using the same test article by making a minor modification to the test article. The rod and two spheres will be deliberately imbalanced causing the rod and two spheres to rotate to a vertical orientation. The imbalance will be measured indirectly from the measured pendulum frequency caused by the imbalance. The conductive coating on the top sphere will be eliminated and a fixed conductive sphere will be placed beside the bottom sphere (attached to the rod). All three conductive spheres will be charged to a voltage \( V_0 \), causing the rod to rotate from the vertical axis to a new equilibrium angle. It is this equilibrium angle that will be measured and predicted. These results will be provided along with the presentation of this paper at the Adaptive Structures Forum to be held in Hilton Head, SC on April 21-22, 1994.

References


