A CONTROL SYSTEM DESIGN APPROACH FOR FLEXIBLE SPACECRAFT

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Abstract

A control system design approach for flexible spacecraft is presented. The control system design is carried out in two steps. The first step consists of determining the "ideal" control system in terms of a desirable dynamic performance. The second step consists of designing a control system using a limited number of actuators that possess a dynamic performance that is close to the ideal dynamic performance. The effects of using a limited number of actuators is that the actual closed-loop eigenvalues differ from the ideal closed-loop eigenvalues. A method is presented to approximate the actual closed-loop eigenvalues so that the calculation of the actual closed-loop eigenvalues can be avoided. Depending on the application, it also may be desirable to apply the control forces as impulses. The effect of digitizing the control to produce the appropriate impulses is also examined.
the ideal dynamic performance will require distributed actuation and sensing devices. On the other hand, it is recognized that the use of these distributed devices is, for the most part, impractical. The second step consists of constructing a control system of minimal cost which exhibits dynamic performance that is as close as possible to the ideal. Therefore, the second step consists of implementing the uniform damping control obtained in the first step using discrete actuation and discrete sensing devices. As it turns out, ideal performances can be obtained with a relatively small number of actuators.

II. **Mathematical Description**

The equations of motion of a flexible structure can be expressed in the form

\[ M\ddot{x}(t) + Kx(t) = F(t) \]  

(1)

where \( x(t) \) is an n-dimensional vector of nodal displacements and slopes and \( F(t) \) are forces and moments at the corresponding nodes. \( M \) and \( K \) denote n by n mass and stiffness matrices, respectively, and overdots represent differentiations with respect to time. The mass and stiffness matrices are obtained using the finite element method. Common computer programs capable of generating the mass and stiffness matrices include NASTRAN and SAP.

Associated with the equations of motion, one commonly defines the eigenvalue problem

\[ \lambda M\phi = K\phi \]  

(2)

The solution of this problem is known as the eigensolution which consists of the eigenvector \( \phi \) and the associated eigenvalue \( \lambda \). There exist n eigensolutions, i.e. n eigenvectors \( \phi_r \) (\( r = 1, 2, \ldots, n \)) and n associated eigenvalues \( \lambda_r \) (\( r = 1, 2, \ldots, n \)). Structural dynamicists commonly refer to
(A) Rigid-body Modal Responses \((\omega_r = 0)\)

We rewrite Equation (5) in the state space by introducing the change of variables \(\tilde{u}_r(t) = [\dot{u}_r(t) \ u_r(t)]^T\) and obtain the modal equations

\[
\dot{\tilde{u}}_r(t) = A\tilde{u}_r(t) + Bf_r(t) \tag{7}
\]

where

\[
A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{8}
\]

The solution to Equation (8) can be converted into a difference equation. Letting \(T\) denote the time step, and letting \(\dot{u}_r(k)\) and \(u_r(k)\) denote the modal velocity and modal displacement at time \(kT\), \((k = 0, 1, 2, \ldots)\) we obtain the difference equations

\[
\dot{u}_r(k+1) = \dot{u}_r(k) + Tf_r(k) \tag{9a}
\]

\[
u_r(k+1) = \dot{u}_r(k)T + u_r(k) + \frac{T^2}{2}f_r(k) \tag{9b}
\]

Equation (9) is used to compute the response of a rigid-body mode.

(B) Flexible-body Modal Responses \((\omega_r \neq 0)\)

Equation (5) describes the motion of an undamped oscillator. However, structures experience small degrees of structural damping. We can introduce some damping into the mathematical model at the exponential rate \(\alpha_r\) by replacing Equation (5) with

\[
\ddot{u}_r(t) + 2\alpha_r \dot{u}_r(t) + (\alpha_r^2 + \omega_r^2)u_r(t) = f_r(t) \tag{10}
\]

The natural frequency in Equation (10) is identical to that in Equation (5).

We rewrite Equation (10) by introducing the change of complex variables

\[
u_r(t) = \text{Re}\{w_r(t)\}, \quad \dot{u}_r(t) = \text{Re}\{\lambda_r w_r(t)\} \quad \text{where} \quad \lambda_r = -\alpha_r + i\omega_r,
\]

and we obtain the complex modal state equations

\[
\dot{w}_r(t) = \lambda_r w_r(t) + f_r(t)/(i\omega_r) \tag{11}
\]
We observe from Equation (15) that only the rth modal displacement and the rth modal velocity control the rth modal force. Such a control is referred to as natural because the modal coordinates do not couple the equations of motion (Refs. 7 and 11). Substituting Equation (15) into Equation (5), we obtain the closed-loop modal equations

\[ u_r(t) + 2 \alpha \dot{u}_r(t) + (\alpha^2 + \omega_r^2) u_r(t) = 0 \quad (r = 1, 2, \ldots, m) \]  

(16)

The corresponding closed-loop eigenvalues are given by

\[ \lambda_{1,2} = \frac{1}{2} \left[ -2\alpha \pm \sqrt{(2\alpha)^2 - 4(\alpha^2 + \omega_r^2)} \right] = -\alpha \pm i \omega_r \]  

(17)

From Equation (17), the closed-loop modes all decay at the same exponential rate \( \alpha \) and the closed-loop frequencies of oscillation are identical to the natural frequencies. Also, observe that the control law, Equation (14), is independent of the spacecraft stiffness. As a general rule of thumb, when a control system is designed to dampen modes in a more non-uniform manner, the control law will tend to depend more on the structural stiffness. Therefore, in the interest of designing a robust control system and one which does not depend explicitly on the fidelity of the mathematical model of stiffness, we uniformly dampen the motion.

The objective to uniformly dampen the motion can also be arrived at from other points of view. For example, let us assume that we wish to drive the motion of a given point on the structure to equilibrium at the exponential rate \( \alpha \), i.e. we wish that a given point be exponentially stable. Then, it can be shown that this point will be exponentially stable at the exponential decay rate \( \alpha \) only if all of the natural modes of vibration are exponentially stable at the rates \( \alpha_r \) not less than \( \alpha \). Also, note that any effort to dampen a given mode at an exponential rate \( \alpha_r \) strictly greater than \( \alpha \) will require unnecessary fuel. Therefore, the most effective way to drive the motion of any point to equilibrium at the exponential decay rate \( \alpha \) is by damping the motion of the natural modes uniformly at the exponential decay rate \( \alpha \) (Ref. 11).
Equation (21) can be rewritten in the form
\[
\ddot{u}_r(t) + 2\alpha \dot{u}_r(t) + (\alpha^2 + \omega_r^2)u_r(t) = \sum_{s=1}^{m} \left( (\psi_r^T \phi_s - 2\alpha \delta_{rs}) \dot{u}_s(t) + (\psi_r^T \phi_s^T - \alpha^2 \delta_{rs}) u_s(t) \right), \quad (r = 1, 2, \ldots, m)
\] (22)
The flexible-body modes and the rigid-body modes in Equation (22) can we rewritten in the state space by introducing the complex change of variables
\[
u_r(t) = \text{Re} \{w_r(t)\}, \quad \dot{\nu}_r(t) = \text{Re} \{\lambda_r w_r(t)\}, \quad (r = 1, 2, \ldots, m)
\] (23)
where \(\lambda_r = -\alpha + i\omega_r\) are the system eigenvalues that would be obtained using the ideal control system. We obtain the complex modal state equations
\[
\dot{\nu}_r(t) = \nu_r(t) + 1/2 \sum_{s=1}^{m} \left( g_{rs} w_s(t) + g_{rs} \dot{w}_s(t) \right)
\] (24)
where
\[
g_{rs} = (\alpha^2 \delta_{rs} - \psi_r^T \phi_s)/i\omega_r + (2\alpha \delta_{rs} - \psi_r^T \phi_s^T) \lambda_s/i\omega_r,
\] (r, s = 1, 2, \ldots, m)
(25)
The eigenvalues of the controlled spacecraft lie in the circles with centers \(C_r\) and associated radii \(R_r\), given by
\[
C_r = \lambda_r + g_{rr}/2, \quad R_r = \sum_{s=1}^{m} \left| g_{rs} \right| - \sum_{s \neq r} \left| g_{rs} \right|
\] (26)
Note that the centers \(C_r\) are also first-order approximations of the eigenvalues associated with the ideal control system. Equation (26) can be used in order to compare the performance of the control system design with the performance of the ideal control system.

IV. Digitization of the Controls

In the previous section, distributed controls were discretized in space leading to the implementation of the controls using a limited number of control forces. The controls acted continuously in time. The controls can also be discretized in time leading to digital controls. In the process, the dynamic performance of the controls are expected to change depending on the
given an initial unit step input at \( x = 4.0 \) for \( 2.0 \) seconds. We design for a uniform exponential decay rate of \( \alpha = 1.0 \) and we assume that 1 percent structural damping is present in the beam.

As a first step, the ideal control system is designed. The free response is shown in Figure 1 and the ideal control system response is shown in Figure 2. The ideal closed-loop eigenvalues are given in Table 1. Next we consider implementing the control system using a discrete number of control forces. In order to approximate the ideal control system, we locate control forces along the beam at the points \( P_r \), \( (r = 1, 2, ..., s; s = 4, 5) \) (See Table 2). The associated control laws are given by

\[
F_r(t) = -2\alpha m_r \dot{x}_r(t) - \alpha^2 m_r x_r(t), \quad m_r = a/s, \quad (r = 1, 2, ..., s) \quad (32)
\]

where \( x_r(t) \) is the displacement at \( P_r \). Here, again, \( m_r \) represents the mass in the region of the \( r \)-th control force. The responses of the beam with the discrete controls are shown in Figures 3 and 4. The corresponding fuels consumed by the controls are shown in Figures 5 and 6. Also, the corresponding first-order approximations of the closed-loop eigenvalues are given in Tables 3 and 4.

Next we digitize the control law Equation (30). The responses of the beam using digitized discrete controls are shown in Figures 7 and 8. The corresponding fuels consumed by the controls are shown in Figures 9 and 10. A computer program listing is given in Appendix A.

VI. Conclusions

A control system design approach for flexible spacecraft has been presented. The control system design is carried out in two steps. The first step consists of determining an "ideal" uniform exponential rate at which we desire the spacecraft motion to dampen. Next, we construct a control with
References


**Locations $P_r$ of the Control Forces**

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Five Forces</td>
<td>1.0</td>
<td>3.0</td>
<td>5.0</td>
<td>7.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Four Forces</td>
<td>2.0</td>
<td>4.0</td>
<td>6.0</td>
<td>8.0</td>
<td>--</td>
</tr>
</tbody>
</table>

*Table 2*
First Order Approximation of the
Closed Loop Eigenvalues Using Four Control Forces

<table>
<thead>
<tr>
<th>r</th>
<th>$\lambda_r = -\alpha_r + i\omega_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.25 + 10.078</td>
</tr>
<tr>
<td>2</td>
<td>-1.25 + 10.747</td>
</tr>
<tr>
<td>3</td>
<td>-1.25 + 11.167</td>
</tr>
<tr>
<td>4</td>
<td>-1.25 + 11.500</td>
</tr>
<tr>
<td>5</td>
<td>-1.25 + 12.760</td>
</tr>
<tr>
<td>6</td>
<td>0.00 + 13.517</td>
</tr>
<tr>
<td>7</td>
<td>-1.25 + 14.810</td>
</tr>
<tr>
<td>8</td>
<td>-1.25 + 16.296</td>
</tr>
<tr>
<td>9</td>
<td>-1.25 + 17.978</td>
</tr>
<tr>
<td>10</td>
<td>0.00 + 19.920</td>
</tr>
</tbody>
</table>

Table 4
Figure 2. Controlled Response-Distributed Forces, Continuous in Time.
Figure 4. Controlled Response—Four Control Forces, Continuous in Time.
Figure 7. Controlled Response—Four Control Forces, Impulses Every 0.2 Seconds.

$\hat{u}(1,t)$  $\hat{u}(3,t)$  $\hat{u}(5,t)$  $\hat{u}(7,t)$  $\hat{u}(9,t)$

$u(1,t)$  $u(3,t)$  $u(5,t)$  $u(7,t)$  $u(9,t)$
Figure 9. Fuel Control Forces, Impulses Every 0.2 Seconds.

Figure 10. Fuel Control Forces, Impulses Every 0.3 Seconds.
Appendix A. Continued.

```fortran
OMEGA = (I*PI/AA)**2
ALFA = 2.*ZETA*OMEGA
LAMDA = (0., 1.)*OMEGA - ALFA
WRITE(11, *) LAMDA
1    CONTINUE
    DO 3 I = 1, N
    DO 2 J = 1, M
      N1 = N + 1
      VEC(J) = SQRT(2)*SIN(J*PI*(I-0.0)/N1)
2    CONTINUE
    WRITE(11, 100)(VEC(J), J = 1, M)
3    CONTINUE
100   FORMAT(2X, 5E15.6)
CLOSE(11)
STOP
END
```

BOTTOM
.NULL.
CCCC
CCCC  CCCCCCCCCCCCCCCCCCCCCCCCC
CCCC  CCCCCCCCCCCCCCCCCCCCCCCCC
CCCC
THE CONTROL PARAMETERS ARE DEFINED
CCCC
CCCC  CCCCCCCCCCCCCCCCCCCCCCCCC
CCCC  CCCCCCCCCCCCCCCCCCCCCCCCC
CCCC
REAL*8 XMASH(9)
OPEN(UNIT=11,FILE='CONTROL',STATUS='UNKNOWN')
OPEN(UNIT=12,FILE='DAT',STATUS='UNKNOWN')
ALFA=1.0
READ(12,*)MMM
READ(12,*)N
DO 10 J=1,N
  XMASH(J)=10. /N
  CONTINUE
10
  CONTINUE
  KTIME=1
WRITE(11,*)(XMASH(I),I=1,N)
WRITE(11,*)(KTIME)
WRITE(11,*)(ALFA)
CLOSE(11)
CLOSE(12)
STOP
END

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Appendix A. Continued.

SUBROUTINE RESP(VEC, VAL, X, XDOT, T, FOR, M, N, U, UDOT, W)

SUBROUTINE RESP

THE SYSTEM RESPONSE IS UPDATED FOR EACH
TIME STEP. THE COMPUTATION DISTINGUISHES
BETWEEN RIGID-BODY MOTION AND FLEXIBLE-BODY
MOTION.

REAL*8 VEC(9, 25), X(9), XDOT(9), FOR(9), U(25), UDOT(25)
COMPLEX*16 VAL(25), W(25), PSI, GAMA, OMI
DO 3 J = 1, M
  F = 0
  DO 1 K = 1, N
    F = F + VEC(K, J) * FOR(K)
  1    IF (CDABS(VAL(J)).LT.1.D-6) GOTO 2
    PSI = CDEXP(VAL(J)*T)
    OM = (0., -1.)*VAL(J)
    OMI = (0., 1.)*OM
    GAMA = (PSI - 1)/VAL(J)/OMI
    W(J) = PSI*W(J) + GAMA*F
    U(J) = W(J)
    UDOT(J) = VAL(J)*W(J)
  3  END
Appendix A. Continued.

PROGRAM

SYSTEM RESPONSE PROGRAM

THE RESPONSE OF THE CONTROLLED SYSTEM IS
COMPUTED AT VARIOUS POINTS.

REAL*8 VEC(9,25),X(9),XDOT(9),FOR(9),U(25),UDOT(25),FOR(9)
REAL*8 XMASS(9)
COMPLEX*16 VAL(25),W(25)
INTEGER IFOR(9)
OPEN(UNIT=11,FILE='DAT',STATUS='UNKNOWN')
OPEN(UNIT=13,FILE='FORCES',STATUS='UNKNOWN')
OPEN(UNIT=14,FILE='OUT1',STATUS='UNKNOWN')
OPEN(UNIT=15,FILE='OUT2',STATUS='UNKNOWN')
OPEN(UNIT=16,FILE='OUT3',STATUS='UNKNOWN')
OPEN(UNIT=17,FILE='OUT4',STATUS='UNKNOWN')
OPEN(UNIT=18,FILE='OUT5',STATUS='UNKNOWN')
OPEN(UNIT=6,FILE='FOR1',STATUS='UNKNOWN')
OPEN(UNIT=7,FILE='FOR2',STATUS='UNKNOWN')
OPEN(UNIT=8,FILE='FOR3',STATUS='UNKNOWN')
OPEN(UNIT=9,FILE='FOR4',STATUS='UNKNOWN')
OPEN(UNIT=10,FILE='FOR5',STATUS='UNKNOWN')
OPEN(UNIT=19,FILE='CONTROL',STATUS='UNKNOWN')


 Appendix A. Continued.

 X(K) = 0
 XDOT(K) = 0
 CONTINUE
 READ(13,*), NP
 READ(13,*) (IFOR(K), K=1,NP)
 DO 4 I = 1,L
 DO 3 K = 1,N
 FOR(K) = 0
 CONTINUE
 READ(13,*), (FOR(IFOR(K)), K=1,NP)
 CALL LAU(FOR, X, XDOT, I, FORT, XMASH, KTIME, ALFA)
 CALL RESP(UEC, VAL, X, XDOT, T, FOR, M, N, U, UDOT, W)
 WRITE(14,100) TM, X(1), XDOT(1)
 WRITE(15,100) TM, X(3), XDOT(3)
 WRITE(16,100) TM, X(5), XDOT(5)
 WRITE(17,100) TM, X(7), XDOT(7)
 WRITE(18,100) TM, X(9), XDOT(9)
 WRITE(6,100) TM, FORT(1), FORT(2)
 WRITE(7,100) TM, FORT(3), FORT(4)
 WRITE(8,100) TM, FORT(5), FORT(6)
 WRITE(9,100) TM, FORT(7), FORT(8)
 WRITE(10,100) TM, FORT(9), FORT(9)
 TM = T + TM
 CONTINUE
 100 FORMAT(F6.3,2E22.13)
 CLOSE(11)
 CLOSE(13)
 CLOSE(14)
 CLOSE(15)
 CLOSE(16)
Appendix A. Continued.

CONTROL ROBUSTNESS PROGRAM

Ideally, a desirable dynamic performance requires distributed sensing and actuation which is for the most part impractical. Therefore, one resorts to finite-dimensional sensing and actuation. This process of going from distributed to discrete is called control discretization. This program looks at the effects of control discretization on the dynamic performance. Toward this end, we look at:

1) Changes in the neighbourhoods of the closed-loop eigenvalues.

2) First-order perturbations of the closed-loop eigenvalues.

REAL*8 VEC(25,25),C(25,25),D(25,25),XMASS(25),RAD(25)
COMPLEX*16 VAL(25),CEN(25),LAM(25),GRS,GJS,GJI,OM(25)
OPEN(UNIT=11,FILE='DAT',STATUS='UNKNOWN')
OPEN(UNIT=12,FILE='CIRCLE',STATUS='UNKNOWN')
OPEN(UNIT=13,FILE='CONTROL',STATUS='UNKNOWN')
READ(11,*)M
GJI=-(C(J,I)*LAM(IS)+D(J,I))/OM(IR)
GJS=GJS+GJI*VEC(I,IS)
4
   CONTINUE
   GRS=GRS+VEC(J,IR)*GJS
5
   CONTINUE
   IF(IR.EQ.IS)GRS=GRS+(2.*LAM(IR)*ALFA+ALFA**2)/OM(IR)
   IF(IR.EQ.IS)CEN(IR)=CEN(IR)+GRS*.5
   IF(IR.NE.IS)RAD(IR)=RAD(IR)+CDABS(GRS)
6
   CONTINUE
7
   CONTINUE
   WRITE(12,100)(LAM(I),I=1,M)
100  FORMAT(2X,'IDEAL EIGENVALUES'/,25(2X,2E15.5/))
   WRITE(12,150)(XMASS(I),I=1,N)
150  FORMAT(2X,'REGIONAL MASSES'/,25(E15.5/))
   WRITE(12,200)
   WRITE(12,300)(CEN(I),RAD(I),I=1,M)
200  FORMAT(2X,'NEIGHBOURHOODS OF THE CLOSED-LOOP'
   1  ', ' EIGENVALUES'/,2X,4X,'CENTERS-FIRST-ORDER APPROX)',5X
   1  ',2X,'RADII'/)
300  FORMAT(2X,2E15.5,5X,E15.5)
   CLOSE(11)
   CLOSE(12)
   STOP
   END

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VIBRATION SUPPRESSION OF PLANAR TRUSS STRUCTURES UTILIZING UNIFORM DAMPING CONTROL

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Displacement vs. Time

Uncontrolled

Configuration 1

Configuration 2

Configuration 3

Configuration 4

Mode  | Configuration 1  | Configuration 2  |
-------|------------------|------------------|
1      | -.034            | -.067            |
2      | -.034            | -.038            |
3      | -.034            | -.047            |
4      | -.033            | -.055            |
5      | -.033            | -.055            |
6      | -.030            | -.050            |
7      | -.029            | -.032            |
8      | -.033            | -.055            |
9      | -.033            | -.055            |
10     | -.028            | -.041            |
11     | -.028            | -.041            |
12     | -.036            | -.072            |
13     | -.033            | -.041            |
14     | -.032            | -.062            |
15     | -.029            | -.056            |
16     | -.031            | -.051            |
17     | -.030            | -.051            |
18     | -.030            | -.051            |
19     | -.026            | -.016            |

*Eigenvalue $\lambda = a + \omega_{r}i$

Mode  | Configuration 3  | Configuration 4  |
-------|------------------|------------------|
1      | -.114            | -.171            |
2      | -.053            | -.053            |
3      | -.074            | -.115            |
4      | -.089            | -.126            |
5      | -.089            | -.126            |
6      | -.055            | -.054            |
7      | -.012            | -.012            |
8      | -.090            | -.090            |
9      | -.090            | -.090            |
10     | -.040            | -.046            |
11     | -.040            | -.046            |
12     | -.128            | -.128            |
13     | -.070            | -.109            |
14     | -.093            | -.093            |
15     | -.055            | -.082            |
16     | -.051            | -.051            |
17     | -.032            | -.032            |
18     | -.033            | -.033            |
19     | -.017            | -.017            |

*Eigenvalue $\lambda = a + \omega_{r}i$

Table 3 Modal Exponential Decay Rates for the Four Controller Configurations.

Fig. 2 Controller Configuration Effects on Transient Response of Mode A.