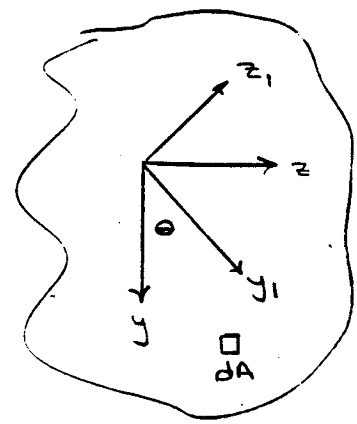
Bending of Nonsymmetric Beams

The moments of inertia for a plane area are given by

$$I_{yy} = \int z^2 dA \qquad I_{zz} = \int y^2 dA \qquad I_{yz} = \int yz dA$$



The transformation formulas for the moments of inertia are

$$I_{y_1y_1} = \frac{I_{yy} + I_{zz}}{2} + \frac{I_{yy} - I_{zz}}{2}\cos 2\theta - I_{yz}\sin 2\theta$$

$$I_{z_1z_1} = \frac{I_{yy} + I_{zz}}{2} - \frac{I_{yy} - I_{zz}}{2}\cos 2\theta + I_{yz}\sin 2\theta$$

$$I_{y_1z_1} = \frac{I_{yy} - I_{zz}}{2}\sin 2\theta + I_{yz}\cos 2\theta$$

The angle to the principal axes of inertia is determined from

$$\theta_p = \frac{1}{2}\tan^{-1}\left(-\frac{2I_{yz}}{I_{yy} - I_{zz}}\right)$$

The generalized flexure formula is

$$\sigma_x = \frac{(M_y I_{zz} + M_z I_{yz})z - (M_z I_{yy} + M_y I_{yz})y}{I_{yy}I_{zz} - I_{yz}^2}$$

while the curvatures are given by

$$\kappa_y = \frac{d^2 v_y}{dx^2} = \frac{M_z I_{yy} + M_y I_{yz}}{E(I_{yy}I_{zz} - I_{yz}^2)}$$

$$\kappa_z = \frac{d^2 v_z}{dx^2} = -\frac{M_y I_{zz} + M_z I_{yz}}{E(I_{yy}I_{zz} - I_{yz}^2)}$$

Note that if the y and z axes happen to be the principal axes, $I_{yz} = 0$ and

$$\sigma_x = \frac{M_y z}{I_{yy}} - \frac{M_z y}{I_{zz}}$$

$$\kappa_y = \frac{d^2 v_y}{dx^2} = \frac{M_z}{EI_{zz}}$$

$$\kappa_z = \frac{d^2 v_z}{dx^2} = -\frac{M_y}{EI_{yy}}$$